## TRANSIENT NATURAL CONVECTION BETWEEN TWO VERTICAL WALLS HEATED/COOLED ASYMMETRICALLY

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The present paper analyses a closed form solution for the transient free convective flow of a viscous and incompressible fluid between two vertical walls as a result of asymmetric heating or cooling of the walls. The convection currents between the walls occur due to a change in the temperature of the walls to that of the temperature of the fluid. The Laplace transform method has been used to find the solutions for the velocity and temperature fields by solving the governing partial differential equations. The numerical values obtained from the analytical solution show that the flow is initially in the downward direction near the cooled wall for negative values of the buoyancy force distribution parameter. The temperature field of both the air and water gradually decreases and becomes negative near the cooled wall for all negative values of the buoyancy force distribution approaches a steady state when the non-dimensional time becomes comparable with the actual Prandtl number.

Key words: transient, natural convection, asymmetric heating, buoyancy force distribution parameter.

#### 1. Introduction

Investigation of the natural convection transport processes due to the coupling of the fluid flow and heat transfer is a challenging as well as interesting phenomenon. It has been extensively studied between vertical walls because of its importance in many engineering applications. Ostrach (1952) has extensively presented steady laminar free-convective flow of a viscous incompressible fluid between two vertical walls. Ostrach (1954) and Sparrow *et al.* (1959) have studied the combined effects of a steady free and forced convective laminar flow and heat transfer between vertical walls. Aung *et al.* (1972) and Aung (1972), Miyatake and Fuzii (1972), Miyatake *et al.* (1973) have presented their results for a steady free convective flow between vertical walls by considering different physical situations of transport processes. Nelson and Wood (1989a, b) have presented in detail the combined effect of heat and mass transfer processes on a free convective flow between vertical parallel plates with symmetric and asymmetric boundary conditions on the plates. Lee (1999) has presented a combined numerical and theoretical investigation of laminar natural convection heat and mass transfers in open vertical parallel plates when unheated entry and unheated exit are present.

Unsteady natural convection flows, where devices are either heated or cooled have its wide applications in technological processes, such as the cooling of the core of a nuclear reactor in the case of power or pump failures and the warming and cooling of electronic components. Kettleborough (1972) has

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described numerically the transient laminar two-dimensional motion of a viscous incompressible fluid between two heated vertical plates in which the motion is generated by a temperature gradient perpendicular to the direction of the body force. The result from a numerical study of the transient natural convection flow between two vertical parallel plates has been presented by Joshi (1988) by considering uniform temperature and a uniform heat flux on the walls. Singh (1988) and Singh *et al.* (1996) have studied the flow behaviour of a transient free convective flow of a viscous incompressible fluid in a vertical channel when one of the channel walls is moving impulsively and the walls are heated asymmetrically. Paul *et al.* (1996) have studied the transient free convective flow in a vertical channel having a constant temperature and constant heat flux on the channel walls. Jha *et al.* (2003) have recently presented an analytical solution for the transient free convection flow in a vertical channel as a result of symmetric heating.

The present study addresses the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls when convection between the vertical parallel walls is set up by a change in the temperature of the walls as compared to the fluid temperature. A non-dimensional parameter is used in order to characterise the vertical wall temperature with respect to the fluid temperature. This parameter also characterises distribution of the buoyancy force between the vertical walls and thus provides a very convenient and generalised framework to study the effects of changing the wall temperature. Numerical calculations of the analytical solutions obtained by the Laplace transform method have been performed in order to study the effects of the buoyancy force distribution parameter. The effect of the Prandtl number and buoyancy force distribution parameter have been shown in the graphical and tabular forms.

#### 2. Mathematical analysis

An unsteady free-convective flow of an incompressible and viscous fluid between two vertical parallel walls is taken here when the temperature of the fluid is different than that of the temperature of the walls. The x'-axis is considered along one of the vertical walls while the y'-axis is perpendicular to it. Initially the temperatures of the fluid as well as the walls are same as  $T'_m$ . At time t' > 0, the temperature of the walls at y' = 0 and H is instantaneously raised or lowered to  $T'_h$  and  $T'_c$  respectively such that  $(T'_h > T'_c)$  which is thereafter maintained constant. Also it is assumed that all the fluid properties are constant except that the influence of the density variation with temperature is considered in the body force term. By defining the non-dimensional variables

$$y = y'/H, t = t'\nu/H^{2}, (2.1)$$
$$u = u'\nu/g \beta(T'_{h} - T'_{m})H^{2}, \theta = (T' - T'_{m})/(T'_{h} - T'_{m}),$$

the momentum and thermal energy conservation equations in a non-dimensional form for the considered model are derived as follows

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \Theta, \qquad (2.2)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2}.$$
(2.3)

The initial and boundary conditions for the velocity and temperature fields in a non-dimensional form are obtained as follows

Additional non-dimensional parameters appearing in the above equations are the Prandtl number Pr and the buoyancy force distribution parameter R defined by

$$\Pr = \mu C_p / k , \qquad R = (T'_c - T'_m) / (T'_h - T'_m).$$
(2.5)

It should be noted that the parameter *R* is very important due to its fundamental effect on the transport processes between the vertical walls. This parameter essentially fixes the orientation of the fluid temperature  $T'_m$  with respect to the vertical wall temperatures  $T'_h$  and  $T'_c$ . The case  $T'_h > T'_m > T'_c$  results for R < 0. When *R* is in the range 0 < R < 1, it results in either  $T'_m < T'_c < T'_h$  or  $T'_c < T'_h < T'_m$ . For R = 0 and 1, the cases  $T'_m = T'_c$  and  $T'_h = T'_c$  are obtained respectively. The symbols used in the above equations are defined in the nomenclature.

An analytical solution of the partial differential Eqs (2.2) and (2.3) under the initial and boundary conditions (2.4) by using the Laplace transform technique is derived as follows

$$u = \frac{1}{(\Pr - I)} \sum_{n=0}^{\infty} \left[ -F(a_1, t, I) + F(a_2, t, I) + F(a_1, t, \Pr) - F(a_2, t, \Pr) + R[F(a_3, t, I) - F(a_4, t, I) - F(a_3, t, \Pr) + F(a_4, t, \Pr)] \right],$$
(2.6)

$$\Theta = \sum_{n=0}^{\infty} \left[ \operatorname{erfc}\left(\frac{a_2 \sqrt{\Pr}}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{a_1 \sqrt{\Pr}}{2\sqrt{t}}\right) + R \left\{ \operatorname{erfc}\left(\frac{a_3 \sqrt{\Pr}}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{a_4 \sqrt{\Pr}}{2\sqrt{t}}\right) \right\} \right]$$
(2.7)

where

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$$a_1 = 2n + 2 - y$$
,  $a_2 = 2n + y$ ,  $a_3 = 2n + 1 - y$ ,  $a_4 = 2n + 1 + y$ .

In the above equations, erfc is the complementary error function defined by

$$\operatorname{erfc}(\eta) = \frac{2}{\sqrt{\pi}} \int_{\eta}^{\infty} \exp(-x^2) dx$$
,

and

$$F(x, y, P) = \left(y + \frac{x^2 P}{2}\right) \operatorname{erfc}\left(\frac{x\sqrt{P}}{2\sqrt{t}}\right) - \left(\frac{x\sqrt{yP}}{\sqrt{\pi}}\right) \exp\left(-\frac{x^2 P}{4y}\right).$$

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Using Eqs (2.6) and (2.7), the skin-friction and the rate of heat transfer on both the walls in a nondimensional form are given by

$$\begin{aligned} \tau_{0} &= \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{2}{\Pr - I} \sum_{n=0}^{\infty} \left[ b_{I} \left\{ \operatorname{erfc}\left(\frac{b_{I}}{\sqrt{t}}\right) - \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{I}\sqrt{\Pr r}}{\sqrt{t}}\right) \right\} + b_{2} \left\{ \operatorname{erfc}\left(\frac{b_{2}}{\sqrt{t}}\right) - \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{2}\sqrt{\Pr r}}{\sqrt{t}}\right) \right\} + \\ &- \sqrt{\frac{t}{\pi}} \left\{ \exp\left(-\frac{b_{I}^{2}}{t}\right) + \exp\left(-\frac{b_{2}^{2}}{t}\right) - \sqrt{\Pr r} \left\{ \exp\left(-\frac{b_{I}^{2}\operatorname{Pr}}{t}\right) + \exp\left(-\frac{b_{2}^{2}\operatorname{Pr}}{t}\right) \right\} \right\} + \\ &+ R \left\{ b_{3} \left\{ -\operatorname{erfc}\left(\frac{b_{3}}{2\sqrt{t}}\right) + \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{3}\sqrt{\Pr r}}{2\sqrt{t}}\right) \right\} + 2\sqrt{\frac{t}{\pi}} \left\{ \exp\left(-\frac{b_{3}^{2}}{4t}\right) - \sqrt{\operatorname{Pr}\exp\left(\frac{b_{3}^{2}\operatorname{Pr}}{4t}\right)} \right\} \right\} \right], \end{aligned}$$

$$(2.8) \\ &\tau_{I} = \frac{\partial u}{\partial y}\Big|_{y=I} = \frac{2}{\Pr - I} \sum_{n=0}^{\infty} \left[ b_{3} \left\{ \operatorname{erfc}\left(\frac{b_{3}}{2\sqrt{t}}\right) - \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{3}\sqrt{\Pr r}}{2\sqrt{t}}\right) \right\} - 2\sqrt{\frac{t}{\pi}} \left\{ \exp\left(-\frac{b_{3}^{2}}{4t}\right) - \sqrt{\Pr r} \operatorname{erfc}\left(\frac{b_{3}^{2}\sqrt{\Pr r}}{4t}\right) \right\} + \\ &+ R \left\{ -b_{I} \left\{ \operatorname{erfc}\left(\frac{b_{I}}{\sqrt{t}}\right) - \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{I}\sqrt{\Pr r}}{\sqrt{t}}\right) \right\} - b_{2} \left\{ \operatorname{erfc}\left(\frac{b_{2}}{\sqrt{t}}\right) - \operatorname{Pr}\operatorname{erfc}\left(\frac{b_{2}\sqrt{\Pr r}}{\sqrt{t}}\right) \right\} + \\ &+ \sqrt{\frac{t}{\pi}} \left\{ \exp\left(-\frac{b_{I}^{2}}{t}\right) + \exp\left(-\frac{b_{2}^{2}}{t}\right) - \sqrt{\Pr r} \left\{ \exp\left(-\frac{b_{I}^{2}\operatorname{Pr}}{t}\right) + \exp\left(-\frac{b_{2}^{2}\operatorname{Pr}}{t}\right) \right\} \right\} \right\} \right]$$

and

$$\operatorname{Nu}_{0} = -\frac{\partial\theta}{\partial y}\Big|_{y=0} = \sum_{n=0}^{\infty} \sqrt{\frac{\operatorname{Pr}}{\pi t}} \left[ \exp\left(-\frac{b_{1}^{2}\operatorname{Pr}}{t}\right) + \exp\left(-\frac{b_{2}^{2}\operatorname{Pr}}{t}\right) - 2R\exp\left(-\frac{b_{3}^{2}\operatorname{Pr}}{4t}\right) \right],$$
(2.10)

$$\operatorname{Nu}_{I} = -\frac{\partial\theta}{\partial y}\Big|_{y=I} = \sum_{n=0}^{\infty} \sqrt{\frac{\operatorname{Pr}}{\pi t}} \left[ 2\exp\left(-\frac{b_{3}^{2}\operatorname{Pr}}{4t}\right) - R\exp\left(-\frac{b_{1}^{2}\operatorname{Pr}}{t}\right) + R\exp\left(-\frac{b_{2}^{2}\operatorname{Pr}}{t}\right) \right]$$
(2.11)

where

$$b_1 = k + 1$$
,  $b_2 = k$ ,  $b_3 = 2k + 1$ .

It should be noted that the above results correspond to Singh *et al.* (1996) and Jha *et al.* (2003) for the cases R = 0 and R = I respectively. To compare the steady state solution obtained from Eqs (2.6) and (2.7) with the exact solution for the steady case, we put  $\frac{\partial}{\partial t} = 0$  in Eqs (2.2) and (2.3) and find the solution of the resulting equations by using only boundary conditions of (2.4). By doing so the solutions for the velocity and temperature fields are obtained as follows

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$$u_{s} = \frac{1}{6} \left[ (1 - R)y^{3} - 3y^{2} + (R + 2)y \right],$$
(2.12)

$$\theta_S = (R-1)y + 1.$$
(2.13)

It has been found that the numerical values of the velocity and temperature fields calculated from the expressions (2.12) and (2.13) have matched very well with the numerical values obtained from the expressions (2.6) and (2.7) at the steady state time. Also the steady state values of the skin-friction and Nusselt number from the expressions (2.8)-(2.11) on both the walls are almost the same as the values obtained from the following expressions

$$\tau_{0_s} = \frac{du_s}{dy}\Big|_{y=0} = \frac{R+2}{6}, \qquad (2.14)$$

$$\mathbf{t}_{I_s} = \frac{du_s}{dy}\Big|_{y=I} = \frac{2R+1}{6}$$
(2.15)

and

Nu<sub>0s</sub>, Nu<sub>1s</sub> = 
$$\frac{d\theta_s}{dy}\Big|_{y=0,1} = I - R$$
. (2.16)

#### 3. Results and discussion

The transport phenomenon occurring as a result of a free convective flow between the vertical walls is governed by the physical parameters R and Pr. The numerical computations carried out from Eqs (2.6) and (2.7) for the velocity and temperature fields are shown in Figs 1 to 5 respectively, whereas, the numerical values of the skin-friction and rate of heat transfer are presented in the tabular form. In the process of calculation, the values of the Prandtl number (Pr) are taken as 0.71 and 7.0 corresponding to the realistic fluids, i.e., air and water respectively.



Fig. 1. Velocity profiles of air (Pr = 0.71) when  $0 \le R \le 1$ .

Figure 1 illustrates the velocity profiles of air for the case  $0 \le R \le 1$ . It is clear from the figure that the velocity of air for R = 0 is maximum near the heated wall and then gradually decreases towards the cooled wall because the temperature of the fluid  $T'_m$  as well as the cooled wall  $T'_c$  is the same and less than the temperature of the heated wall  $T'_h$ . The maximum velocity occurs in the middle (y = 0.5) for R = 1 and it shifts towards the wall y = 0 as R changes from 1 to 0. This phenomenon occurs because for R > 0, the temperature of both the walls  $T'_h$  and  $T'_c$  is greater than the fluid temperature  $T'_m$ , while for R = 1, the temperature of both the walls  $T'_h$  and  $T'_c$  is the same but greater than the fluid temperature  $T'_m$  as a result of which a symmetric flow occurs between the wall, and thus, the nature of the flow is of parabolic type. A close study of the figure indicates that the buoyancy force distribution parameter R plays a major role in controlling the velocity profiles and enhancement in the fluid velocity is due to the increase in the values of R. The velocity is time dependent and the steady state occurs as the non-dimensional time approaches the value of the Prandtl number.

In Fig.2, the velocity profiles of air have been shown for the case R < 0. It can be observed that the upward flow occurs near the heated wall for all considered values of R and t. Further, we can observe that at t = 0.3, the reverse flow does not occur near the cooled wall for R = -0.3 and -0.5 whereas it occurs for R = -0.7 as a result of more cooling of the air. The magnitude of the upward flow increases near the heated wall while the magnitude of the downward flow decreases near the cooled wall as time increases. Finally at the steady state, the upward flow has been formed near the heated wall and the downward flow near the cooled wall for R = -0.7 only. The reason is that for negative values of R, the fluid temperature  $T'_m$  is greater than the temperature of the cooled wall  $T'_c$  while less than the temperature of the hot wall  $T'_h$ . The magnitude of the upward flow near the cooled wall increases as R changes from -0.3 to -0.7. Velocity profiles of water are not given here for the sake of brevity as they are almost of the same nature as those of air.



Fig.2. Velocity profiles of air (Pr = 0.71) when R < 0.

Figures 3 and 4 show the influence of the buoyancy force distribution parameter R on the temperature profiles of the air and water respectively for the case  $0 \le R \le 1$ . These figures show that in the initial state the temperature of the water is less than that of the air. This difference can be explained by the fact that at large values of the Prandtl number the thermal diffusivity is small, so the penetration of the thermal energy in the case of the water is less compared to the air. The temperature of both air and water increases with time and attains its steady state value as the non-dimensional time approaches the value of the Prandtl number of the corresponding fluids. The steady state temperature of both air and water is the same and directly proportional to R. Enhancement in the temperature of both air and water is due to an increase in the values of R.



Fig.3. Temperature profiles of air (Pr = 0.71) when  $0 \le R \le 1$ .



Fig.4. Temperature profiles of water (Pr = 7.0) when  $0 \le R \le 1$ .

In Fig.5, we have plotted the temperature profiles of both air and water with respect to time *t* for the case R < 0. A study of the curves shows that at t = 0.1, the temperature of water is same upto y = 0.6 for all considered values of *R* (curves 2, 7, 12) and changes beyond it. We can conclude that initially the temperature of water does not change in the flow domain near the heated wall as *R* changes. The temperature of both air and water is time dependent and the steady state occurs as the non-dimensional time approaches the value of the Prandtl number of the corresponding fluids.



Fig.5. Temperature profiles of air (Pr = 0.71) and water (Pr = 7.0) when R < 0.

The numerical values of skin-friction  $\tau_0$  and  $\tau_1$  as well as the Nusselt number Nu<sub>0</sub> and Nu<sub>1</sub> are presented in Tabs 1 and 2 for the air and water respectively for different values of the buoyancy force distribution parameter *R* and time *t*. An important observation from the tables is that the numerical values of skin-friction at both the walls increase as the values of the buoyancy force distribution parameter *R* increases. The numerical values of  $\tau_0$  and  $\tau_1$  are same for R = I because a symmetric flow occurs for this case. The values of  $\tau_0$  are always greater than  $\tau_1$  for other values of *R*. Initially on both the walls, the numerical values of skin-friction of air is greater than for water while the Nusselt number of air is less than water. As *R* changes from *I* to -0.7, differences between the values of  $\tau_0$  and  $\tau_1$  increase, which suggests that a desired flow formation can be obtained by assigning a suitable value to *R*. It is clear from the tables that the heat transfer rate on both the walls decreases as the value of *R* increases for both the air and water. The rate of heat transfer on both the walls at the initial state is greater than that of the steady state for all considered values of *R* except for R = 0. It can be observed that the steady state values of Nu<sub>0</sub> are exactly the same as Nu<sub>1</sub> for all values of *R* and that is also confirmed by Eq.(2.16).

Table 1. Numerical values of skin-friction and the Nusselt number of air (Pr = 0.71).

R	t	$ au_{O}$	$\tau_I$	Nu <sub>0</sub>	Nu <sub>1</sub>
-0.7	0.1	0.170682	-0.102812	1.862516	1.563647
	0.3	0.208110	-0.075218	1.709269	1.690731
	0.5	0.215301	-0.067193	1.700575	1.699425
	0.7	0.216465	-0.066866	1.700036	1.699964
	<i>S.S</i>	0.216666	-0.066668	1.700000	1.700000

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-0.5	0.1	0.177217	-0.064101	1.760600	1.262485
	0.3	0.235741	-0.014254	1.515448	1.484552
	0.5	0.247725	-0.002275	1.500958	1.499042
	0.7	0.249665	-0.000333	1.500059	1.499941
	<i>S.S</i>	0.250000	-0.000000	1.500000	1.500000
-0.3	0.1	0.183752	-0.025390	1.658684	0.961322
	0.3	0.263372	0.046708	1.386878	1.278373
	0.5	0.280148	0.063481	1.301342	1.298658
	0.7	0.282865	0.066199	1.300083	1.299917
	<i>S.S</i>	0.283333	0.066666	1.300000	1.300000
0.0	0.1	0.193555	0.032675	1.505810	0.509579
	0.3	0.304819	0.138154	1.030897	0.969103
	0.5	0.280148	0.162116	1.001917	0.998083
	0.7	0.332664	0.165998	1.000119	0.999881
	<i>S.S</i>	0.333334	0.166666	1.000000	1.000000
0.3	0.1	0.203357	0.090742	1.352937	0.057836
	0.3	0.346265	0.229600	0.740165	0.659834
	0.5	0.377418	0.260751	0.702491	0.697508
	0.7	0.382464	0.265798	0.700154	0.699845
	S.S	0.383333	0.266665	0.700000	0.700000
0.5	0.1	0.209892	0.129453	1.2514021	-0.243325
	0.3	0.373896	0.290564	0.546344	0.453655
	0.5	0.409841	0.326508	0.502874	0.497125
	0.7	0.415664	0.332331	0.500178	0.499821
	<i>S.S</i>	0.416666	0.333332	0.500000	0.500000
07	0.1	0.216428	0.168164	1.149105	-0.544488
	0.3	0.401527	0.351528	0.352524	0.2477475
	0.5	0.442264	0.392264	0.303258	0.296742
	0.7	0.448863	0.398863	0.300202	0.299798
	S.S	0.450000	0.400000	0.300000	0.300000
1.0	0.1	0.226230	0.226230	0.996231	0.996231
	0.3	0.442973	0.442973	0.061793	0.061793
	0.5	0.490899	0.490899	0.003832	0.003832
	0.7	0.498663	0.498663	0.000237	0.000237
	<i>S.S</i>	0.500000	0.500000	0.000000	0.000000
ole 2. Numer	rical values of sl	kin-friction and the	Nusselt number of	water ( $Pr = 7.0$ ).	
R	t	$ au_{O}$	$\tau_{I}$	Nu <sub>0</sub>	Nu <sub>1</sub>
-0.7	0.3	0.152021	-0.094246	2.736467	1.923667
	1.0	0.198993	-0.083625	1.858579	1.565587
	3.0	0.215634	-0.067698	1.708733	1.691267
	7.0	0.216662	-0.066670	1.700031	1.699969
	S.S	0.216666	-0.066666	1.700000	1.700000

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-0.5	0.3	0.156792	-0.060501	2.733275	1.378608
	1.0	0.220824	-0.028545	1.754821	1.266501
	3.0	0.248279	-0.001720	1.514555	1.485445
	7.0	0.249993	-0.000005	1.500052	1.499948
	<i>S.S</i>	0.250000	-0.000000	1.500000	1.500000
-03	03	0 161564	-0.026757	2 730083	0 833549
0.0	1.0	0.242655	0.026535	1 651064	0.967415
	3.0	0 280984	0.064258	1 320377	1 279623
	7.0	0.283324	0.066658	1.300073	1.299928
	S.S	0.283333	0.066667	1.300000	1.300000
0.0	03	0 168722	0 023859	2 72 52 95	0 015960
0.0	1.0	0 275402	0.010915	1 495428	0 518787
	3.0	0.329892	0.016322	1.029110	0.970890
	7.0	0.333320	0.016665	1.000104	0.999896
	<i>S.S</i>	0.333333	0.166666	1.000000	0.9999999
03	03	0 175880	0 074476	2 720506	-0 801627
0.0	1.0	0 308149	0 191776	1 339792	0 701587
	3.0	0.378860	0.262193	0.737842	0.662157
	7.0	0.383317	0.266650	0.7001.34	0.699865
	S.S	0.383333	0.266666	0.700000	0.699999
0.5	0.3	0.180652	0.108220	2.717314	-1.346686
	1.0	0.329980	0.246856	1.236034	-0.228926
	3.0	0.411505	0.328172	0.543664	0.456335
	7.0	0.416648	0.333315	0.500155	0.499844
	<i>S.S</i>	0.416666	0.333333	0.500000	0.499999
0.7	0.3	0.185424	0.141965	2.714122	-1.891745
	1.0	0.351811	0.301937	1.132277	-0.528012
	3.0	0.444150	0.394150	0.349486	0.250513
	7.0	0.449979	0.399979	0.300175	0.299824
	<i>S.S</i>	0.450000	0.400000	0.300000	0.299999
1.0	0.3	0.192581	0.192581	2.709334	2.709334
	1.0	0.384558	0.384558	0.976640	0.976640
	3.0	0.493118	0.493118	0.058219	0.058219
	7.0	0.499975	0.499975	0.000206	0.000206
	<i>S.S</i>	0.500000	0.500000	0.000000	0.000000

#### 4. Conclusions

A transient free convection flow of a viscous and incompressible fluid between two vertical parallel walls occurring as a result of asymmetric heating/cooling of the walls has been studied by defining a buoyancy force distribution parameter R. It has been found that the symmetric/asymmetric nature of the flow formation

can be obtained by giving a suitable value to the buoyancy force distribution parameter. Formation of the upward flow occurs near the heated wall while the downward flow forms near the cooled wall for negative values of the buoyancy force distribution parameter.

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#### Nomenclature

- Cp specific heat at constant pressure  $J Kg^{-l}K^{-l}$
- g acceleration due to gravity  $ms^{-2}$
- H distance between two vertical walls m
- k thermal conductivity  $Wm^{-1}K^{-1}$
- $Nu_0$  Nusselt number at the wall y = 0
- $Nu_1$  Nusselt number at the wall y = 1
  - Pr Prandtl number
  - R buoyancy force distribution parameter
  - t time in non-dimensional form
  - t' time s
- $T'_c$  temperature of the wall at y' = H, K
- $T'_h$  temperature of the wall at y' = 0, K
- $T'_m$  initial temperature of the fluid *K* 
  - u fluid velocity in non-dimensional form
- u' velocity of the fluid  $m s^{-1}$
- y dimensionless co-ordinate perpendicular to the walls
- y' co-ordinate perpendicular of the walls *m*
- $\beta$  coefficient of thermal expansion  $K^{-1}$
- $\mu$  dynamic viscosity of the fluid  $kg m^{-1}s^{-1}$
- $\theta$  temperature of the fluid in non-dimensional form
- v kinematic viscosity of the fluid  $m^2 s^{-1}$
- $\tau_0$  skin-friction in non-dimensional form at the wall y = 0
- $t_1$  skin-friction in non-dimensional form at the wall y = 1

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