# AIR LIFT II. TWO PHASE FLOW IN THE MIXING AREA OF THE AIR LIFT*

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***Abstract***
*In the paper authors have written the momentum equations and the continuity equations in differential form for describing the solid particle parameters in the mixing area of the air lift. Numerical solution of the equations, and presentation of the diagrams, obtained on the basis of a given example.*

**Nomenclature**

| | |
|---|---|
| $A_o$ [m$^2$] | particle cross section |
| $b$ [m] | height of beginning of the pipe over distribution layer |
| $C_D$ [-] | drag coefficient |
| $d$ [m] | diameter |
| $d_o$ [m] | diameter of solid particle |
| $F$ [N] | drag force |
| $F_f$ [N] | friction force |
| $f$ [-] | friction factor |
| $G_1$ [N] | weight of a particle |
| $g$ [m/s$^2$] | acceleration of gravity |
| $m_1$ [kg] | mass of one particle |
| $\dot{m}_{gk}$ [kg/s] | mass flow of the conveying gas flowing out of the nozzle |
| $p$ [Pa] | pressure |
| $p_o$ [Pa] | atmospheric pressure |
| $p_c$ [Pa] | pressure on the place of © |
| $p_k$ [Pa] | pressure of the gas at outlet of the nozzle |
| $p_j$ [Pa] | pressure under the nozzle |
| $R$ [J/kgK] | gas constant |
| $Re$ [-] | Reynolds number |
| $R_p$ [m] | Radius of the conveying pipe |
| $v$ [m/s] | velocity |
| $v_{gc}$ [m/s] | Gas velocity on the place of © |
| $v_{gy}$ [m/s] | Gas velocity on the place of „y" |
| $v_j$ [m/s] | velocity under the nozzle |
| $v_k$ [m/s] | velocity at the outlet of nozzle |
| $v_m$ [m/s] | material velocity |
| $v_{my}$ [m/s] | material velocity on the place of „y" |
| $v_{mo}=\varphi\, v_m$ [m/s] | sedimentation velocity in the tank |

**Greek letters**

| | |
|---|---|
| $\alpha$ [degree] | angle |
| $\mu$ [kg/ms] | absolute viscosity of the air |
| $\rho_g$ [kg/m$^3$] | gas density |
| $\rho_{gy}$ [kg/m$^3$] | gas density at the place of „y" |
| $\rho_{go}$ [kg/m$^3$] | gas density at atmospheric pressure |
| $\rho_m$ [kg/m$^3$] | material concentration |
| $\rho_{my}$ [kg/m$^3$] | material concentration on the place of „y" |

**Indices**

| | |
|---|---|
| $f$ | friction |
| $g$ | gas |
| $m$ | solid material |
| $p$ | pipe |
| ©, $j$, $k$, $y$ | mark of place |
| 1 | one particle |

*Paper of AIR LIFT I. see REFERENCES [1]

**Mixing of air and solid particles in the starting section**

The values of solid mass flow „$\dot{m}_m$" and gas mass flow passing through the air distribution layer and flowing towards the nozzle can be read at place $R = R_p$ in Figures 5, 6 and 7 of paper [1].

Gas mass flow „$\dot{m}_{gk}$" leaving the nozzle mixes with gas mass flow „$\dot{m}_g$" flowing towards the nozzle and material mass flow „$\dot{m}_m$" in the „$b$" wide layer over the distribution layer within the initial section (starting section) of the vertical conveying pipe. The solid-air mixture continues to flow upward in the vertical conveying pipe after exchanging their momentums (see Figure 1). The following assumptions are made for setting up the equations:

- The tightening effect caused by the particles in the control volume will not be taken into account
- The change in the state of the gas is considered to be isometric
- Velocity „$v_m$” and concentration „$\rho_m$” are considered to be constant for the material flowing upwards within the volume of rotation bordered by the „$y_o$” high triangle in Figure 1.
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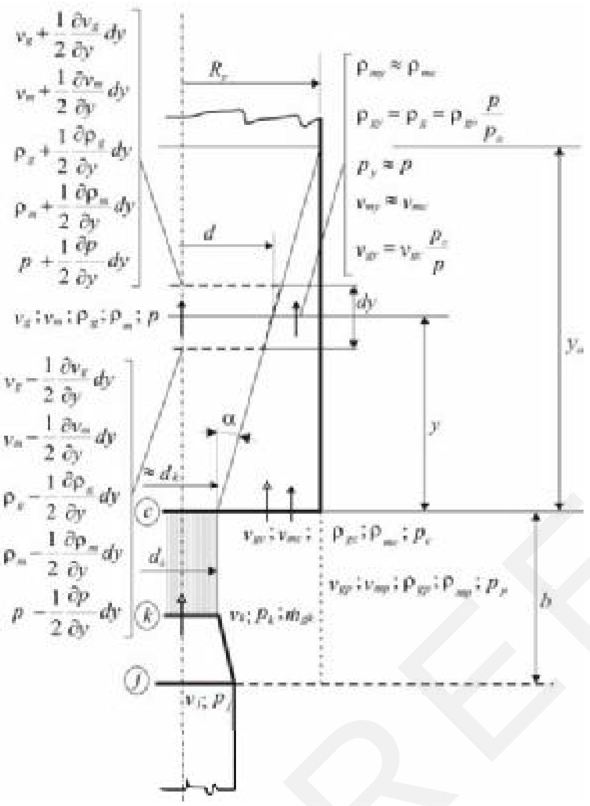

**Fig. 1 Starting section of conveying pipe of air lift**

**Continuity equation for gas**

The geometrical parameters of the control volume marked with dashed line in the sketch shown in Figure 1 are as follows:

$$\begin{aligned}
& d_{y+dy/2} = d + tg\alpha\, dy \\
& A_{y+dy/2} = (d + tg\alpha\, dy)^2 \pi / 4 \\
& d_{y\ \ dy/2} - d - tg\alpha\, dy \\
& A_{y-dy/2} = (d - tg\alpha\, dy)^2 \pi / 4 \\
& d = 2tg\alpha\, y + d_k
\end{aligned} \qquad (1)$$

At the beginning of the starting section in cross-section © the gas and material mass flow values are identical with values $p=p_c=p_k=p_p$ , „$\rho_{mp}$”, „$v_{mp}$”, „$\rho_{gp}$”, „$v_{gp}$” assumed at place $R = R_p$ and calculated with equations 4, 10, 12, 17, 19 in paper [1]. Thus, the following relationship can be written:

$$\begin{aligned}
& 2 R_p \pi\, b\, \rho_{mp} v_{mp} = \pi \left( R_p^2 - \frac{d_k^2}{4} \right) \rho_{mc} v_{mc} \\
& 2 R_p \pi\, b\, \rho_{gp} v_{gp} = \pi \left( R_p^2 - \frac{d_k^2}{4} \right) \rho_{gc} v_{gc}
\end{aligned} \qquad (2)$$

Using equations 2, the values of „$v_{mc}$” and „$v_{gc}$” at the beginning of the starting section (in cross-section „c”) can be calculated with the assumption of

„$\rho_{mp} \approx \rho_{mc}$” and „$\rho_{gp} \approx \rho_{gc}$”.

Thus, the continuity equation can be written in the following form:

$$\begin{aligned}
& A_{y-dy/2} \left( \rho_g - \frac{d\rho_g}{2} \right) \left( v_g - \frac{dv_g}{2} \right) + \\
& + \left( A_{y+dy/2} - A_{y-dy/2} \right) \rho_{gy} v_{gy} - \\
& - A_{y+dy/2} \left( \rho_g + \frac{d\rho_g}{2} \right) \left( v_g + \frac{dv_g}{2} \right) - 0
\end{aligned} \qquad (3)$$

After the transformation of this equation, and neglecting second-order members as well as taking $\rho_{gy}v_{gy}=\rho_{gc}v_{gc}$ and $\rho_{gc} = \rho_{go} \frac{p_c}{p_o}$ into consideration, the continuity equation of gas will be:

$$\frac{dp}{dy} = \frac{p_c v_{gc}}{d\, v_g} 4tg\alpha - \frac{p}{d} 4tg\alpha - \frac{p}{v_g} \frac{dv_g}{dy} \qquad (4)$$

**Continuity equation for solid particles**

The continuity equation in this case is:

$$\begin{aligned}
& A_{y-dy/2} \left( \rho_m - \frac{d\rho_m}{2} \right) \left( v_m - \frac{dv_m}{2} \right) + \\
& + \left( A_{y+dy/2} - A_{y-dy/2} \right) \rho_{my} v_{my} - \\
& - A_{y+dy/2} \left( \rho_m + \frac{d\rho_m}{2} \right) \left( v_m + \frac{dv_m}{2} \right) = 0
\end{aligned} \qquad (5)$$

After transformation and rearranging, and neglecting second-order members as well as taking approximations $v_{my} \approx v_{mc}$ and

$\rho_{my} \approx \rho_{mc}$ into consideration, the following equation is obtained:

$$\frac{d\rho_m}{dy} = \frac{\rho_{mc} v_{mc}}{d\, v_m} 4tg\alpha - \frac{\rho_m}{d} 4tg\alpha - \frac{\rho_m}{v_m} \frac{dv_m}{dy} \quad (6)$$

As can be seen, continuity equations 3 and 5 are identical in their format.

**Momentum equation for solid particles**
The momentum equation can be written in the following form:

$$- A_{y\ dy/2}\left(\rho_m - \frac{d\rho_m}{2}\right)\left(v_m - \frac{dv_m}{2}\right)^2 -$$
$$- \left(A_{y+dy/2} - A_{y-dy/2}\right)\rho_{my} v_{my}^2 +$$
$$+ A_{y+dy/2}\left(\rho_m + \frac{d\rho_m}{2}\right)\left(v_m + \frac{dv_m}{2}\right)^2 =$$
$$= dF - dG_m - dF_f \quad (7)$$

Taking into consideration that

$$dF = \frac{\pi d^2}{8 m_1} \frac{\rho_{go} p}{p_o} \rho_m A_o C_D \left(|v_g| - |v_m|\right)^2 dy \quad (8)$$

and

$$dG_m + dF_f - (1+f)\rho_m \frac{d^2 \pi}{4} dy\, g \quad (9)$$

after transformation and neglecting second-order members as well as taking equations 6, 8; 9 into consideration, the following relationship is obtained:

$$\frac{dv_m}{dy} = -\frac{\rho_{mc} v_{mc}}{d\, \rho_m} 4tg\alpha + \frac{\rho_{mc} v_{mc}^2}{\rho_m v_m d} 4tg\alpha +$$
$$+ \frac{\rho_{go} p}{2 m_1 p_o} A_o C_D \frac{\left(|v_g| - |v_m|\right)^2}{v_m} - \frac{g}{v_m}(1+f) \quad (10)$$

**Momentum equation for gas**
Using the symbols of Figure 1, the theorem of momentum can be written in the following form:

$$- A_{y\ dy/2}\left(\rho_g - \frac{d\rho_g}{2}\right)\left(v_g - \frac{dv_g}{2}\right)^2 -$$
$$- \left(A_{y+dy/2} - A_{y\ dy/2}\right)\rho_{gy} v_{gy}^2 +$$
$$+ A_{y+dy/2}\left(\rho_g + \frac{d\rho_g}{2}\right)\left(v_g + \frac{dv_g}{2}\right)^2 -$$
$$- A_{y-dy/2}\left(p - \frac{dp}{2}\right) - A_{y+dy/2}\left(p + \frac{dp}{2}\right) +$$
$$+ \left(A_{y+dy/2} - A_{y-dy/2}\right)p - dF \quad (11)$$

After transformation and neglecting second-order members as well as taking $v_{gy} = v_{gc}\frac{p_c}{p}$ and $\rho_{gy} - \rho_g = \rho_{go}\frac{p_c}{p_o}$ into consideration, the following relationship is obtained:

$$\frac{dv_g}{dy} = \frac{v_g}{\left(\frac{\rho_{go} v_g^2}{p_o} - 1\right) pd} \Bigg[ 4ptg\alpha - 4tga\frac{p_c^2 v_{gc}}{p\, v_g} -$$
$$- 4tg\alpha\, \rho_{gc} v_{gc} v_g \frac{p_c}{p} + 4tg\alpha \frac{\rho_{gc} v_{gc}^2 p_c}{p_o} -$$
$$- \frac{\rho_{go} \rho_m pd}{2 m_1 p_o} A_o C_D \left(|v_g| - |v_m|\right)^2 \Bigg] \quad (12)$$

**Diagrams obtained by solving the equations**
Equations 3, 5, 9, 11 have been solved using the numerical method of Runge-Kutta.

The initial data of the solution can be read at the $R = R_p$ values of Figures 5, 6, 7 in paper [1]. These have been converted to the initial values taken at the place marked © ($y$=0) of the starting section. These values are: $v_{gc} \approx 6.27$ m/s; $p_c$=133.23 kPa.

Further data:
α = 7°; $v_k$ = 100 m/s; $d_k$ = 0.085 m;
$\dot{m}_{gk}$ = 0.91 kg/s; $v_j$ = 25 m/s; $p_j$ = 145.3 kPa;
$p_c$ = 133.23 kPa;
$\rho_{mp} = \rho_{mc}$ = 202 kg/m$^3$; $R_p$ = 0.125 m;
$\rho_{gc}$ = 1.6 kg/m$^3$; $\dot{m}_{gk} + \dot{m}_{gc}$ = 1.34 kg/s;
$d_o$=1.5*10$^{-4}$ m; solid material: fly ash.

Figure 2 shows the diagrams of gas velocity „$v_g$” and material velocity „$v_m$”.

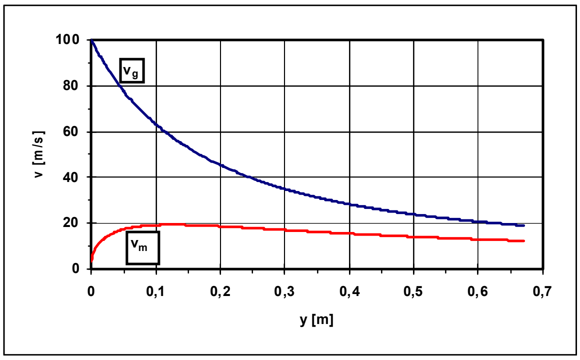


**Fig. 2 Gas- and solid material velocity in the starting section of conveying pipe**

Figure 3 shows the diagrams of material concentration „$\rho_m$" and pressure „$p$".

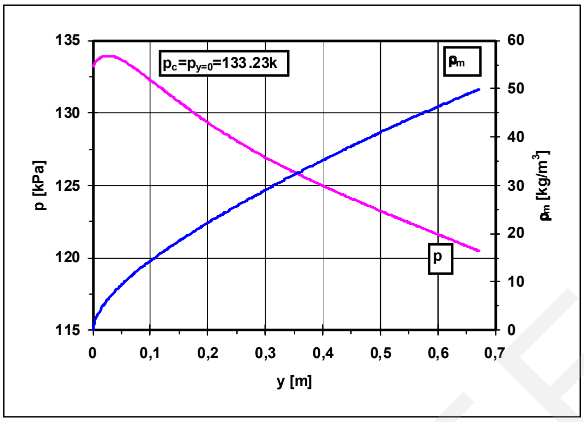


**Fig. 3 Material concentration and pressure in the primer flow in the starting section of conveying pipe**

**Solids-air mixture flowing in the vertical pipe after the starting section**

Figure 4 shows the material parameters inside the connected vertical section in knowledge of the parameters at the end of the acceleration section. These diagrams have been calculated with computer program developed for calculation the various parameters of the pipe in the Department of Hydrodynamic Systems.

Input data of pipe:

$v_g$=18.92 m/s gas velocity
$v_m$=12.25 m/s material velocity
p=120.5 kPa pressure
$\rho_m$=49.9 kg/m$^3$ concentration

Figure 4 shows the variations of gas velocity „$v_g$", material velocity „$v_m$" and pressure „$p$" inside the vertical pipe.

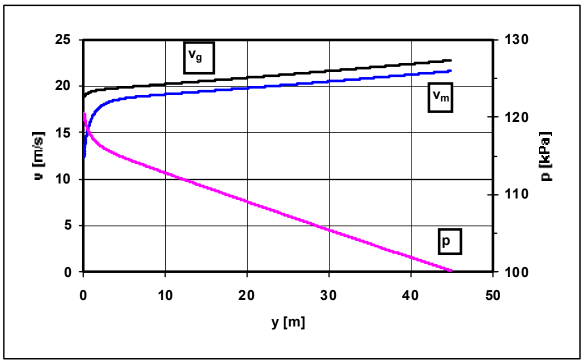


**Fig. 4 Velocity and pressure distribution along the vertical pipe length**

The following pipe-end data can be read from the charts:

$y_p$ = 45 m vertical conveying distance
$v_g$ = 22.8 m/s pipe-end gas velocity
$v_m$ = 21.6 m/s pipe-end material velocity
$p$ = 100 kPa pipe-end pressure

**Conclusions and remarks**

The model presented in this paper is suitable for taking the first step for the description of the complex flow taking place in air lifts. The equations derived allows the main parameters to be calculated and the main dimensions of the equipment to be determined.

The following steps need to be performed in order to verify the model used for the description of the two-phase flow taking place in the mixing zone.

- Determine the value of the angle marked „$\alpha$" in Figure 1.

In addition to the example presented here, the effect of changing the most important parameters involved in the description of two-phase flow taking place in air lift tanks is discussed in detail by paper [2].

**References**


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