POST DOCTORAL RESEARCH

MULTI-PERIOD SHORT-RANGE PRODUCTION PLANNING FOR CEMENT QUARRY OPERATIONS

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CHAPTER 1: INTRODUCTION

1.1. Abstract

The limestone quarry is the major source of raw materials for the cement manufacturing operation. Cement production involves the processing of raw materials that contain - SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, LOI (loss of ignition), SO_3, K_2O, Na_2O, TiO_2, P_2O_5, and Cl, etc. However, depending upon the available reserves, some additives such as sandstone, fly ash, iron ore, and clay are also mixed with limestone to achieve proper blend acceptable to the plant. During production stage of a cement quarry, required percent content of chemicals in the raw mix may only be achieved through the analysis of alternative quarry plans with the objective to select the one requiring the fewest purchased additives from the market.

One of the managerial objectives of a cement manufacturing operation is to minimize the cost of raw materials by satisfying both quantity and quality requirements. Blending of various raw materials to meet strict quality constraints is the basic requirement to accomplish this objective. A linear programming blending optimization model is presented as a short term planning tool, which addresses the objective and constraints of the cement manufacturing operation. The benefits of the model are established in a case study of an existing cement manufacturing operation in the northern part of Pakistan. This application has not only promised a significant cost saving in the provision of raw materials by satisfying quality constraints but also better coordination and engineering control among various departments.

1.2. Background

The prerequisite for installation of a cement manufacturing operation is the availability of raw materials containing required quantity and quality of oxides of calcium, silica, aluminum and iron i.e. CaO, SiO_2, Al_2O_3, Fe_2O_3. Limestone is an industrial mineral which primarily contributes these desirable constituents. However, it also contains some undesirable constituents including MgO, SO_3, K_2O, Na_2O, TiO_2, P_2O_5, and Cl, etc. Therefore, their percent content in raw materials is maintained below the limits dictated by the cement plant. Hence, a limestone quarry becomes the main source of raw materials for cement manufacturing operations; however, in order to meet the strict quality requirements of the cement plant, it is mandatory to blend raw materials from quarry with additives such as sandstone, fly ash, iron ore, and clay, etc. usually purchased from the market (Kathal and Mukherjee, 1999).

In cement quarry operations, the quality of the limestone mined at a given period is solution to the efficiency of cement production. Once the blasted rock from the quarry enters the crushing system in a cement plant it cannot be removed. Therefore, if poor quality rock is placed in the system, an unmarketable product is the result (Kathal and Mukherjee, 1999).

A simple layout of the cement manufacturing operation consists of four steps including (Austin, 1984):

1. Mining of raw materials from the limestone quarry,
2. Developing a raw mix consisting of raw materials from the limestone quarry and additives from the market,
3. Processing (burning) of the raw mix in a cement kiln to produce a product called "clinker", and
4. Grinding of the clinker for distribution in different forms to the customers as cement.

Therefore, a cement plant consists of a series of processes connected by material conveying systems, literally "garbage in, garbage out". The development of raw mix is dependent upon the quality of limestone mined at a given period. However, a few rock units in a quarry contain suitable constituents to run alone. Therefore, the normal process requires blending of high and low-grade material in the quarry and, if required, with the additives from the market (Austin, 1984). In order to reduce the cost incurred on the purchase of additives from the market, limestone mined in a given period must meet the quality/raw mixing constraints such that the required percent content of vital chemical constituents in the raw materials is achieved (Asad, 2001).

Cement demand has increased manifold due to sustained growth and prosperity in the society. Roughly, one hundred and fifty (150) countries are producing cement around the globe. Fig. 1.1 presents a profile of the worldwide cement production with a 75% increase from 1995 to 2006 (USGS, 2007). Annual production of cement in Pakistan was about 22.50 million tons in the year 2006-07 as compared to approximately 8 million tons in 1995-96. Fig. 1.2 presents the profile of cement production in Pakistan. The installed production capacity will possibly expand to 28.21 million tons per annum by the year 2007-2008 with an investment of over one billion dollars (State Bank of Pakistan, 2007).

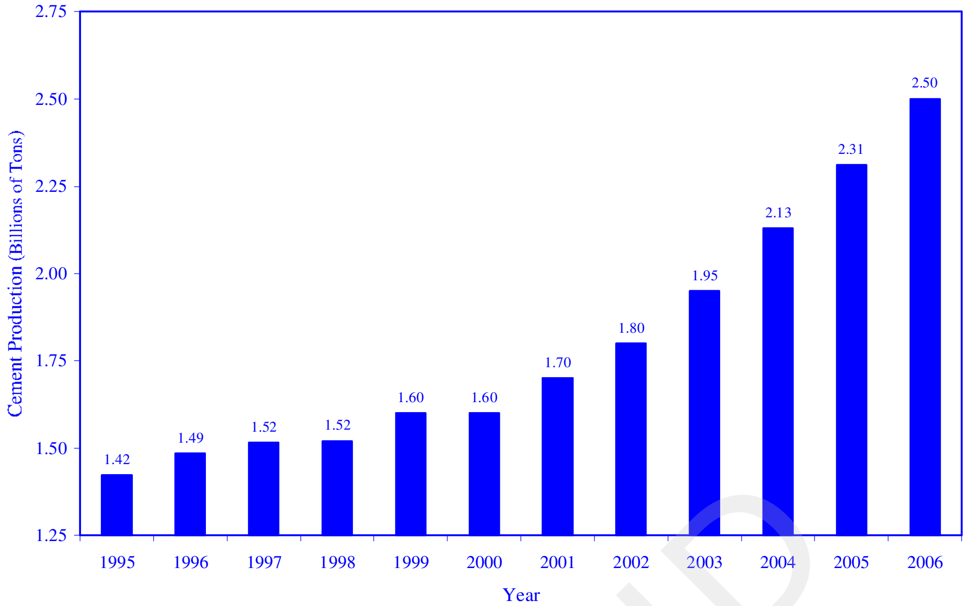

Fig. 1.1: A profile of worldwide cement production from 1995-2006

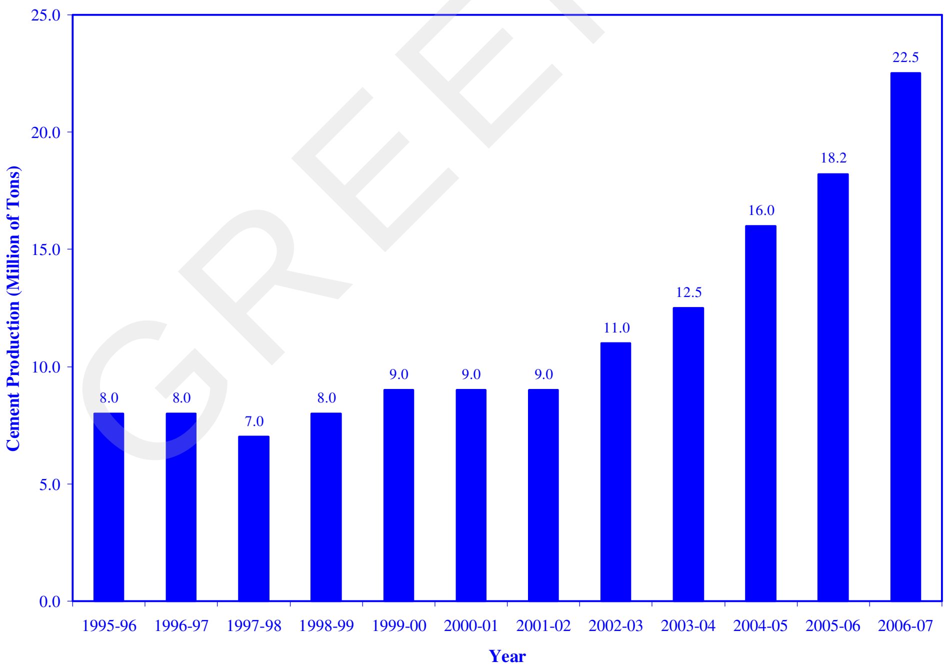

Fig.1.2: A profile of cement production in Pakistan from 1995-2007

Keeping in view the demand, cement industry has witnessed enormous improvements in processing technology to produce low cost product. Currently, it has realized the potential impact of cost savings in producing raw materials from quarry operations. It is focusing on minimization of the cost of raw materials production. Approximately, 1.6 tons of raw materials are required to produce one ton of cement (Austin, 1984; Carr, 1994). Hence, a mere cost saving of $1.00 per ton in producing raw materials will lead to a cumulative saving of billions of dollars.

1.3. Proposed research

Recognizing the possibility of contribution to the cement industry, this research focuses on the development of a cost effective short-range production planning model for the provision of raw materials to the cement plant.

1.3.1. Problem statement

Cement manufacturing requires blending of raw materials from limestone quarry operations, which is dependent upon the knowledge of quality and quantity of mineable reserves.

According to the current practices in Pakistan, the blending of raw materials is achieved through a trial and error procedure. This leads to lack of coordination between the cement quarry and the quality control departments, and in other words, it causes mismanagement and inefficient use of valuable reserves.

1.3.2. Problem solution

The development of short-range production plan for cement quarry operations, which ensures optimum blending of raw materials from the limestone quarry with some of the additives purchased from market. This will help in strategic decision making with respect to production planning and adequate engineering and operational control.

At present, production planning and raw materials blending are accomplished through manual interaction, by trial and error (Dagdelen 1985; Dagdelen and Asad, 2002). Two downsides of the trial and error approach are (Asad 2001; Baumbartner 1989):

1. Failure to analyze alternative plans and selection of the optimum.
2. Inability to perform optimum long-term quarry planning.

The need to plan and operate limestone quarries with optimum production planning has been obvious for sometime. However, with few exceptions, majority of the studies on production planning and blending of raw materials have been conducted with a focus on open pit mining operations, primarily applied to mining of metallic ores. Some of the frequently referred studies include Dagdelen and Johnson (1987), Fytas et al. (1987), Mann and Wilke (1992), Tolwinski and Underwood (1992), and Denby et al.(1998).

The solution to cement quarry production planning is different from metallic ores as the objective is the provision of a proper blend of raw materials to the cement plant (Asad, 2001). In cement quarry operations block model, a block consists of percent content of SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, LOI, SO_3, K_2O, Na_2O, TiO_2, P_2O_5, and Cl, etc. However, in open pit mining operations for metallic ores, a block is assigned an economic value. Hence, in open pit mining operations, a block is ore (valuable) if its economic value is positive or a waste block if its economic value is negative. However, in cement quarry operations, even if a block is low in CaO, it could be high enough in SiO_2 or Al_2O_3, hence, becomes a candidate for mining and processing in the plant. Furthermore, a block located in the overburden benches could still be processed in the plant, if it consists of clay, which is a source of SiO_2, Al_2O_3, and other raw materials (Asad, 2001). This establishes the need to develop a production planning tool exclusively addressing the requirements of cement manufacturing operations.

1.3.3. Research objective

To develop a cost minimization linear programming (LP) based mathematical model for optimum blending of raw materials for short range planning of cement quarry operations.

1.3.4. Research methodology

Following steps shall lead towards the accomplishment of research objectives:

1. Data acquisition from one of the cement manufacturing operation in Pakistan.
    a. Available reserves including quantity and quality of the raw materials in various zones/benches of the limestone quarry.
    b. Production capacity of limestone quarry operation.
    c. Plant production capacity.
    d. Quality and cost of additives mixed with raw materials from limestone quarry operations.
    e. Availability of additives in terms of quantity (tons).

   f. Raw mix/blending (raw materials + additives) requirement for cement manufacturing.
2. Development of LP models for short range planning.
   a. Objective function: minimize the cost of cement quarry operations
   b. Constraints:
      i. Available reserves (block model).
      ii. Production capacity of limestone quarry operation.
      iii. Plant production capacity.
      iv. Quality and cost of additives mixed with raw materials from limestone quarry operations.
      v. Availability of additives in terms of quantity (tons).
      vi. Raw mix/blending (raw materials + additives) requirement for cement manufacturing.
3. Solution of LP models in Microsoft Excel Solver, sensitivity and comparative analysis of models, and preparation of results in presentable format.
4. Report writing.

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CHAPTER 2: CEMENT RAW MATERIALS

2.1. Introduction

Cement manufacturing operations require raw materials of sufficient purity with uniform composition for producing high quality Portland cement. Availability of raw materials in the immediate vicinity of the manufacturing operation is a prerequisite. The primary constituents of Portland cement are silicates of lime. Raw materials composition and proportion not only varies from site to site, but also within a given resource substantial variations are frequent. Nevertheless, any raw material that provides silica and calcium in required composition and proportions is suitable for cement manufacturing. Primarily, raw materials consist of basic calcium carbonate ($CaCO_3$) and the acidic oxides of silica (SiO_2), alumina (Al_2O_3), and iron (Fe_2O_3).

Due to variations in quality of the raw materials, a combination or blend of multiple materials is obligatory to produce ordinary or special types of Portland cement. Therefore, a great variety of naturally occurring minerals sometimes mixed with industrial byproducts are utilized as cement raw materials across the globe.

2.2. Classification of raw materials (Khattak, 2007)

A general classification of main raw materials used for the production of Portland cement includes:

1. Calcareous materials (high $CaCO_3$ content)
2. Argillaceous materials (high SiO_2 content)
3. Correcting materials (to balance the oxides i.e. CaO, SiO_2, Al_2O_3, Fe_2O_3)
4. Other raw materials such as Gypsum

The existence of calcareous and argillaceous materials in abundance identifies the potential location for cement manufacturing operation, because they have proven to provide the sustained supply of raw materials for producing Portland cement, the world over. Table 2.1 presents the approximate percent content of the primary constituent i.e. CaCO3 in the naturally occurring calcareous and argillaceous materials.

Table 2.1: General classification of cement raw materials

Raw material	CaCO3 content
High percentage limestone	95-100%
Marley limestone	85-95%
Lime marl	75-85%
Marl	40-75%
Clay marl	15-40%
Marley clay	5-15%
Clay	<5%

2.2.1. Calcareous materials

Calcareous materials include naturally occurring limestone, calcite, chalk, marble, cement rock, lime marl (a natural mixture of limestone with low percentage of clayey substance), and oyster and seashells.

2.2.1.1. Limestone

Limestone is a sedimentary rock. The term limestone applies to all carbonate rocks containing more than 85% of calcium carbonate. Therefore, it is composed primarily of calcium carbonate with varying minor amounts of magnesium, clay, and sand as impurities. Most of the limestone used for the manufacture of Portland cement are either chemically precipitated or formed due to organic action on drainage waters. Upon reaching the sea, some of the dissolved calcium carbonate is re-precipitated due to its lower solubility in sea water. Surface evaporation and temperature changes may reduce the carbon dioxide content of the water as a result of which calcium carbonate is precipitated from saturated conditions. The limestone so formed is of purely chemical origin. A common variety of limestone in this category is oolitic limestone, which are very pure and composed of so-called ooloths i.e. more or less spherical rock particles grown by accumulation around a nucleus. The limestone quarried at Portland, England belonged to this particular category.

In the other process, accumulation and lithification of fragments of calcareous materials originally secreted from water by marine organism plants and animals takes place. When they die their calcareous remains accumulate at the bottom of the sea as a sedimentary deposit. Many species of algae shellfish and number of creatures living in sea or fresh water build their hard part out of calcium carbonate present there and thus remove the larger part of calcium carbonate. Different forms of limestone such as shelly limestone and coral limestone exists.

There are no vigorous specifications for cement grade limestone. All types of limestone can be used for the manufacture of Portland cement, the higher the purity, the better it is for cement production. The chemical and physical properties of limestone vary considerably, due to nature and presence of impurities. Limestone has a fine grained crystalline structure. Its hardness depends upon the geological age varying between 1.8 and 3.0 on Mohr's scale of hardness. Its specific gravity is 2.6 to 2.8. Only the purest varieties of limestone are white. Clayey substances or iron compound present in limestone influence its color.

2.2.2. Argillaceous materials

Argillaceous materials include naturally occurring clay, china clay, kaolin, shale, and slate stone.

2.2.2.1. Clay

This is another important raw material for the manufacture of Portland cement. It is also a sedimentary rock and clays are formed by the weathering of alkali and alkaline earth containing aluminum silicates, feldspar, and mica. Clay, slate, and shale are of about the same composition but of different ages and different stages of consolidation. Each may be used in manufacturing of Portland cement.

2.2.2.2. Shale

Shale is argillaceous sedimentary rock derived from silt or clay deposited in water in thin layers and subjected to some pressure and cementation with some lithification. Shale is plastic clay rock splitting along its bedding plane. It is almost identical with clay in chemical composition being a clay in a solidified form. Like clay the shale shows wide variations in mineralogical and chemical composition and may occur as hard and dense rock. Shale may contain some sand and that is called sandy shale.

2.2.2.3. Slate stone

Slate is fine grained metamorphic rock derived from the argillaceous ones such as clay, mud stone, and shale. The metamorphism is carried out to such an extent that the original planes of stratification are completely obliterated and new well defined planes called cleavages planes are developed in the rock. Slates are of various colors including grey, purple, or reddish brown. Slates offer a good abrasive resistance.

2.2.3. Correcting materials

In case the desired chemical composition is not achieved with the above mentioned categories of raw materials then a small amount of supplementary or corrective materials are added either individually or jointly depending upon the lacking constituents so as to correct any marginal deviations. These materials are usually added in about 3.5% of the total amount of the raw materials.

2.2.3.1. Silica minerals

Silica is present in the raw materials as silicate (clay marl) or quartz (sand). Mixes which are best for burning are those in which silica is chemically combined, while those where it contains quartz is the worst. Usually sandstone, quartz, diatomite or other form of naturally occurring silica are added to increase the silica content.

2.2.3.2. Bauxite

The term bauxite is applied to rocks or earthy deposits in which the main constituent is alumina. Bauxite is a rock of varying composition and contains different amounts of hydrous aluminum oxides, silica, and small amounts of hydrous iron oxide minerals. Bauxite ore normally occurs in three different forms in nature i.e. gibbsite $Al_2O_3.3H_2O$ (tri-hydrates), boehmite, and disspore $Al_2O_3.H_2O$ (neon-hydrates), thus, bauxite is a term for a family of ores rather than a substance of one definite composition. Bauxite is used to increase the alumina content in the raw materials.

2.2.3.3. Laterite

Laterite is a sedimentary rock comprising a mixture of various minerals. Generally the term laterite describes a mixture rich in the oxides and hydroxides of iron, alumina, and titanium. It is low in silica, magnesia, and alkalies. It shows characteristic red color due to the presence of considerable amount of ferrous minerals. It is used to make up the deficiency of ferric oxide in the raw materials.

2.2.4. Gypsum

Gypsum is found in nature in dehydrate ($CaSO_4$.$2H_2O$), Hemi-hydrate ($CaSO_4.1/2H_2O$) and anhydrite ($CaSO_4$). Gypsum or its derivatives are added to the finished product from cement kiln i.e. clinker during grinding stage. The main purpose is to retard the quick setting tendency of ground cement clinker due to very high reactivity. Gypsum is also reported to act as grinding aid.

2.3. Summary

Various raw materials discussed in chapter supply the basic oxides of calcium, silica, aluminum, and iron ($CaCO_3$ SiO_2, Al_2O_3 and Fe_2O_3). As mentioned earlier, approximately 1.6 tons of raw materials are required to produce one ton of clinker. This includes 1.2-1.3 tons of calcareous material and the remaining is argillaceous material or combination of siliceous materials, bauxite and iron ore. Approximately 0.05-0.06 tons of gypsum is required to inter-grind with one ton of clinker for controlling the setting of Portland cement.

CHAPTER 3: CEMENT MANUFACTURING PROCESS

3.1. Introduction

This research involves the development of a new tool for short term planning of cement quarry operations for the provision of raw materials to the plant. It is, therefore necessary to study the manufacturing process in detail so that the influence of the best production plan, which satisfies the raw mixing constraints, is identified. As mentioned in chapter 1, a simple layout of the cement manufacturing operation consists of four steps i.e. mining of raw materials from limestone quarry, developing a raw mix consisting of raw materials from the limestone quarry and additives from the market, processing (burning) of the raw mix in a cement kiln to produce a product called "clinker", and grinding of the clinker for distribution in different forms to the customers as cement. Fig. 3.1 presents a layout of the cement manufacturing operation.

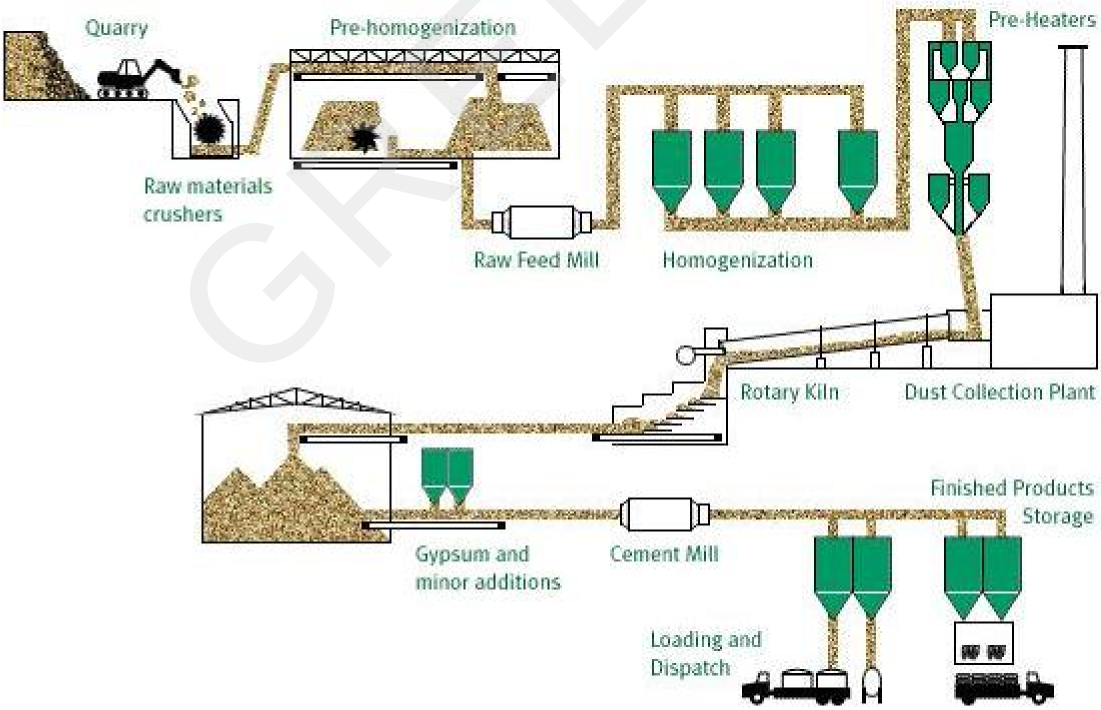


Fig. 3.1: A layout of the cement manufacturing operation

Therefore, a cement manufacturing operation has two major divisions. First is the recovery of raw materials; second is the processing of those materials. The processes are interlinked by material conveying systems, including:

1. Limestone Mining (Quarrying)
    a. Crushing and sizing
2. Processing
    a. Pre-homogenization and raw milling (Grinding)
    b. Burning
    c. Finish grinding

These processes are discussed in detail in the following sections.

3.2. Limestone mining (Quarrying)

Limestone, a source of lime (CaO) is the primary raw material used in cement production. Most cement plants depend upon surface mining or quarrying of limestone. Quarrying is also a mean of extracting other raw materials, such as shale, sandstone, etc. Usual steps in quarry operations include clearing and stripping, drilling, blasting, loading, hauling, and reclamation.

The clearing operation involves removal of vegetation or topsoil from the area. Any overburden is then stripped to uncover the mineral deposit and transported to a disposal area. The stripping and mining are conducted from a single bench or a sequence of benches. Drilling and blasting operations achieve rock breakage. The broken rock is loaded into hauling units for transportation to the crushing plant. Exceptions to this sequence can be found in various operations, and the relative cost of each step may differ from one operation to another. Some quarries have so little overburden that stripping is unnecessary. In other quarries as much as six tons of overburden must be stripped for every ton of limestone recovered. Drilling and blasting may be required to break up hard rock in conventional limestone quarries. These steps however may be eliminated in other quarries because ripping and/or scooping operations can recover softer materials. Loading is accomplished by use of a variety of equipment ranging from electric or diesel shovels to numerous types of loaders. Truck hauling is the most common means of transporting stone out of or within the quarry, although belts, skip hoists, rail cars and wheeled loaders (pick and carry use) are also important for transporting material to the crusher.

3.2.1. Crushing and sizing (Austin, 1984)

Fig. 3.2 presents crushing and sizing stage of cement manufacturing process. In contrast to preparing various size grades of crushed stone aggregates, the objective in processing cement raw materials is to arrive at the fine kiln feed sizes as quickly and economically as possible. Drilling and blasting are done so as to achieve size reduction results consistent with the objectives of fines production and economy. In many quarries, through selective mining care is taken to recover the raw materials in distinct chemical grades. This approach maximizes the recovery and minimizes undesirable impurities. Outside these objectives, quarry practice is governed largely by considerations of the community environment. Therefore, the use of explosives however is limited by safety, noise, and vibration standards. The work to be done is size reduction, and the processes through which this work is applied are a matter of individual plant design and practice. In the cement industry, quarrying extends from the breaking of ground to the delivery of quarried materials to the crushing department and then crushing of raw materials itself. In many quarries, blending for quality control begins with the selection of working faces since the chemistry of the stone in each face is known to some extent.

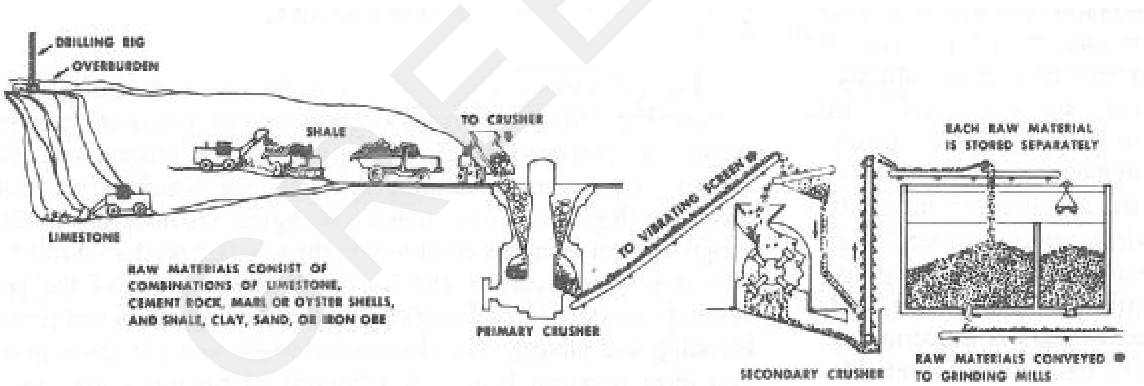


Fig. 3.2: Crushing and sizing (Austin, 1984)

The crushing process is expected to produce material of a specified maximum size. Screens generally determine this crusher-run size, which typically ranges from (0 – ¾") and (0 – 3") mm. Primary and secondary crushing stages and occasionally a tertiary crushing stage generate product for the mills. Virtually all the varieties of crushing machines found in crushed stone plants are also found in the crushing departments of cement plants. Size grading is done mainly to recycle coarse fractions for further crushing, to remove properly sized material from the stream and move it on to the next process.

3.3. Processing (Austin, 1984)

The raw materials needed for cement production are processed in the plant. There are four methods of processing based on the classification of the kiln system (burning step).

1. Long dry process
2. Long wet process
3. Pre-heater process
4. Pre-calciner process

Long rotary kilns have a burner at one end; the raw mix is added at the other end. These kilns are used for both dry and wet processes. Dry-process plants handle all their material in dry form, i.e. raw mix enters the kiln as a dry powder. Wet-process plants add about 32% water to their raw mill, creating slurry that is then pumped to the kiln, where it is dried. Tower kilns have a tall vertical structure at the feed end of the kiln. This structure contains a number of cyclones or stages, which are used to preheat the material. Raw mix is added at the top of the tower and only dry raw mix is used. There are two types of tower kilns: pre-heater and pre-calciner. Pre-heater kilns burn 100% of their fuel inside the kiln at the hot end, with no fuel being burned in the tower. Pre-calciner kilns burn 30% to 50% of their fuel in the tower itself using a second burner in a large chamber called the pre-calciner. The pre-calciner technology offers reduced energy related costs; therefore, the modern cement plants have adopted this technology. As such, wet process has been completely replaced with dry process for burning of cement raw mix.

The cement production process changes the materials physically and chemically as they progress from their raw condition to the finished product. Physical changes include size reduction and proper blending. Chemical changes include calcining calcium carbonate ($CaCO_3$) into calcium oxide (CaO) in a manner promoting lime reaction with other oxide components. In this way, the synthetic silicate minerals and glasses comprising clinker are formed.

3.3.1. Pre-homogenization and raw milling

Fig. 3.3 presents the pre-homogenization and raw milling step of the cement manufacturing operation. Raw materials used for cement manufacturing are normally highly variable in composition. Therefore, it is mandatory to introduce uniformity by mixing the raw materials and additives to achieve proper blend and keep in stockpiles called mixed bed either longitudinally or circular.

A blending bed consists of two stockpiles one of which is built up while the other is being reclaimed and passed on to the grinding units. This particular step is called pre-homogenization or raw mixing of the raw materials.

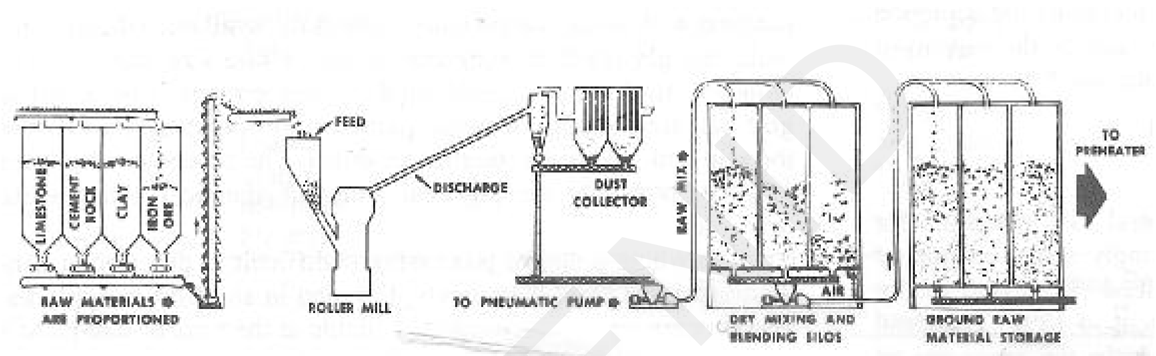


Fig. 3.3: Processing – blending and raw milling (Austin, 1984)

The primary objective of milling is to prepare the approximate sizes and mixtures of raw mix for effective and economic kiln feed processing. Standards of maximum size and percent through a given screen (e.g., 90% through a 200-mesh screen with none coarser than 50-mesh) are set for the finished raw mill product. These standards are usually empirical.

Historically, early plants were dry process. As the technology progressed, a number of factors led to the development of wet-process plants, prime among them being the efficiency of wet blending. Wet marl and some other raw materials can be processed more conveniently without a drying state. For many years the wet process dominated the design of new plants. Recent upward surges of fuel costs provide strong incentives for designing new plants as dry process. Fairly recent developments in dry-grinding mills and heat recuperating pre-heaters, along with the efficient dry blending systems currently available, go far toward removing historical objections (high cost, wear, etc.) to dry process installations.

3.3.2. Burning

Fig. 3.4 presents the raw mix burning process. The Portland cement principally consists of four components: tricalcium silicate ($3CaO.SiO_2$), C_3S, dicalcium silicate ($2CaO.SiO_2$), C_2S, tricalcium Aluminate ($3CaO.Al_2O_3$), C_3A, and a phase approximating to tetra calcium aluminoferrite ($4CaO.\ Al_2O_3.Fe_2O_3$) ,C_4AF. The compounds are formed during burning process inside the kiln by a series of reactions at temperature rising to the region of 1300^o to 1500^{o}C between lime (CaO) on one hand and silica (SiO_2), alumina (Al_2O_3), and iron oxide (Fe_2O_3) on the other. The lime is obtained by decarbonating the calcareous materials such as limestone ($CaCO_3$); the alumina, silica, and iron oxide are obtained by heating argillaceous materials such as clay, shale, or schist. Optimum cement quality is obtained when required proportions of four oxides (CaO, SiO_2, Al_2O_3, and Fe_2O_3) are consistent throughout the cement.

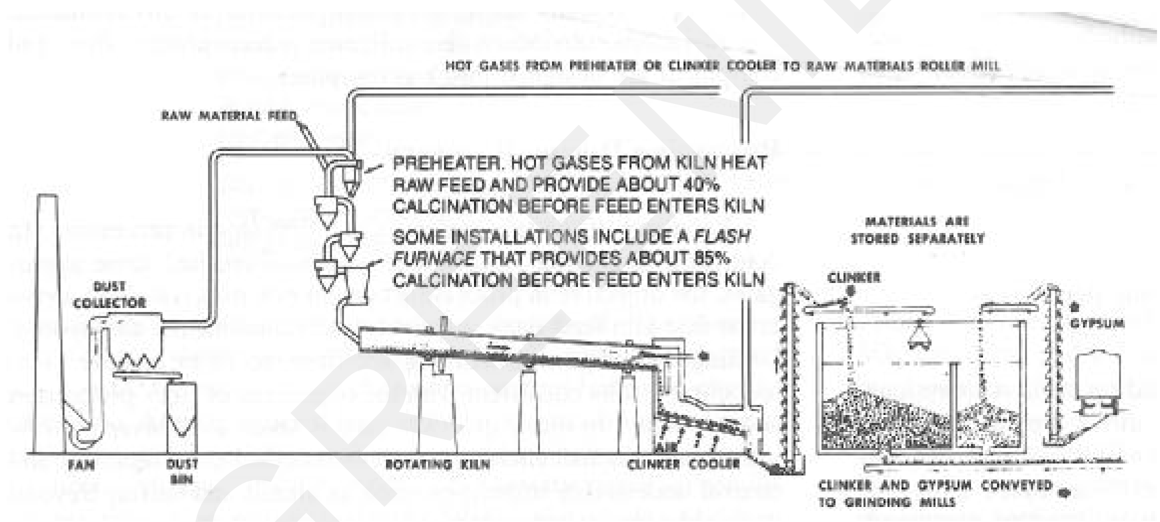


Fig. 3.4: Processing – preheating and burning (Austin, 1984)

The basic steps in the heat treatment are boiling off any slurry water (wet process) at up to 100^{o}C, decarbonation of the calcium carbonate ($CaCO_3$) at up to 1000^{o}C, and heating the decarbonated feed to 1300^o-1500^{o}C according to its composition and fineness, maintaining this temperature sufficiently long for cement compounds to form, and finally cooling the resulting clinker.

The burning of cement begins as a series of reactions between blended solids, and it is only in the later stages of burning that any liquid is formed, causing reactions that produce the cement compounds to take place rapidly. The clinkering process is dependent on three factors i.e. the chemical composition of the raw mix, the physiochemical state of raw mix, and the temperature and period of burning.

Table 3.1 presents the summary of chemical reactions inside the cement kiln.

Table 3.1: Summary of chemical reactions inside the cement kiln (Austin, 1984)

Temperature (°C)	Process	Thermal Change
100°	Evaporation of free water	Endothermic
500° and above	Evolution of combined water from clay minerals	Endothermic
900° and above	Crystallization of amorphous dehydration products of clay Decomposition of $CaCO_3$	Exothermic Endothermic
900° – 1200°	Reaction between CaO and clay	Exothermic
1250° – 1280°	Beginning of liquid formation	Endothermic
Above 1280°	Further liquid formation and completion of formation of cement compounds (C_3S, C_2S, C_3A, and C_4AF)	Probably Endothermic on balance

The product at clinkering temperature consists essentially of crystals of C_3S and C_2S, formation of which is largely completed at this stage, together with a liquid containing CaO with all or most of the Al_2O3, Fe_2O_3, but relatively little SiO_2. The aluminate and ferrite phases therefore form during cooling. The Alumina and iron oxide are the main fluxes in cement burning; without them the silicates could only be formed at much higher temperature or in longer times. The silica ratio (SiO_2 / (Al_2O_3 + Fe_2O_3)) plays an important role in this regard.

The reactivity of raw materials depends not only on its chemical composition, but also on the mineral composition and the size of particles. If material introduced into the kiln is not sufficiently mixed, the reaction will be incomplete. This usually results in the lowering of C_3S content, C_2S and CaO being formed instead in different parts of the clinker nodules.

Burning is the key process in the manufacture of cement. Burning at high temperatures causes properly constituted and prepared raw materials to react and combine to produce clinker containing a balance of synthetic compounds (C_3S, C_3A, C_2S, and C_4AF) that are ground into the desired cement. Worldwide practice includes vertical kiln installations of many varieties and a number of other types of plants. The kiln and its clinker cooler are the heart of the cement plant. Basically everything else is sized to serve it, and it should run at capacity. Plant capacity expansion or reduction implies changes in kiln capacity.

Rotary kilns are steel cylinders, ranging up to 19 feet (6 m) diameter and 754 feet (230 m) in length and lined with fairly thick refractories (mostly brick). A coating of adhering raw materials is developed on refractories, especially in the hot zone, so that the finished materials are processed over similar materials and protection of refractories. Kilns are inclined at four to five feet from the horizontal, so that their rotation (usually at 70 to 90 revolutions per hour in long-wet or dry kilns and 120 to 180 revolutions per hour in pre-heater kilns) moves the materials being processed from the feed end to the discharge end at the desired rate of speed. Retention time of the processed material is two to three hours in a long-wet or dry kiln and 15 minutes in a pre-heater kiln. A balanced, smooth operation is an objective in kiln operation. The calciner and pre-heater kiln system, which is far more stable than other types, has solved many processing problems, and resulted in much improved refractory life.

Fuel is introduced into the rotary kiln under slight pressure through a burner pipe. The burner pipe is positioned in the product discharge end of the kiln, and the fuel blown through the pipe is ignited into a flame, which extends well up into the kiln. The thermal inertia in the burning zone is enormous and results in a stable flame even when burning low-volatility fuels. For economic reasons the preferred kiln fuel is powdered coal and coke. Either fuel oil or natural gas may be used depending on cost and availability. Several kilns use wastes as fuel, particularly wet-process kilns. Other ignitable wastes such as whole or shredded tires, rice hulls, and nutshells are also used. The driving force behind the use of alternative fuels is economics.

As the materials being processed move from the feed end of the kiln toward and into the zone of the flame, they pass through a series of stages. These stages include drying to lose water, calcination of the carbonates (i.e., the driving off of CO_2), and clinkering or burning. In the drying zone, the remaining hot gases in the kiln heat up incoming raw material. In both long-wet and dry kilns, chain sections are used as heat exchangers. The chain links are alternately heated by the hot gases and then heat the raw materials as the kiln rotates. In the calcining zone, raw material is heated to about 1000^{o}C, at which temperature the carbon dioxide is driven off from the limestone to form lime (CaO). This is necessary in order to provide the main chemical needed for the subsequent formation of cement clinker.

The calcined raw material then moves into the clinkering zone. This zone is the heart of the pyroprocess, in which the incipient fusion, partial melting, and reaction to form clinker compounds takes place. Reaction (burning) temperatures proceed to 1300^{o}C (with the formation of tricalcium aluminates) and reach 1500^{o}C (with the formation of dicalcium silicates and tricalcium silicates). The clinker then exits the kiln.

Gas exit temperatures from long-dry process kilns average around 850 to 900^{o}C, being considerably higher than exit temperatures from wet kilns, which range from 200 to 300^{o}C. The thermal economy of dry kilns is thus improved by the addition of a pre-heater. Exit gas temperatures from pre-heater kilns vary from 500^{o}C (2-stage pre-heater) to 275^{o} (6-stage pre-heater). Gases must be above 275^{o}C to ensure effective drying of raw materials in combination dry grinding mills such as the vertical roller mill.

Successful kiln burning produces rounded clinkers (approximately 3 to 20 mm). Clinker leaving the kiln is typically at or above 1000^{o}C and it is cooled in either a reciprocating grate cooler or a planetary cooler. There are other wide varieties of ways to cool clinker, from primitive pits to rotary units to the sophisticated systems. The preferred technique is the reciprocating grate cooler in which moderate pressure air (up to 1100 mm water gauge) is forced up through the deep (900 mm) bed of hot clinker as it slowly moves over the perforated plates. Two major objectives of clinker cooler design is the efficient reuse of the escaping heat and the proper cooling of the clinker to achieve good quality. Appropriate cooling and tempering of clinker is critical in order to achieve subsequent cement strengths. Improper cooling can result in the formation of undesirable clinker compounds with little, if any, hydraulic properties. The pre-heating of combustion air is achieved in the hot end of the grate clinker cooler. Modern designs have greatly improved fuel efficiency. A development of the 1980s has been the cooling of waste air from the clinker cooler in air-to-air heat exchangers prior to passage to dust removal systems. Several plants also recirculate this cooled and cleaned air to the undergrate fans, thereby eliminating an emission point and achieving a small improvement in heat recuperation.

3.2.3. Finish grinding

Fig. 3.5 presents the finish grinding process of cement clinker. Beyond the cooler, conveyors generally move clinker into storage, where it may be segregated, tested, blended, and moved into bins for feeding to the cement (finish) mills. Both the type of clinker storage available and the timing of clinker grinding have quality implications. Clinker that was ground hot produces poor quality cement, as does old clinker that has become hydrated.

The SO_3 content of finished cement is critical in providing the desired setting time for concrete made with that cement. Gypsum (three to six percent) has been inter-ground with clinker to provide SO_3. Anhydrite and other sources of SO_3 are satisfactory substitutes for some, but not all, of this gypsum. Clinker grinding is usually accomplished in rod, ball, vertical roller, and ball race mills.

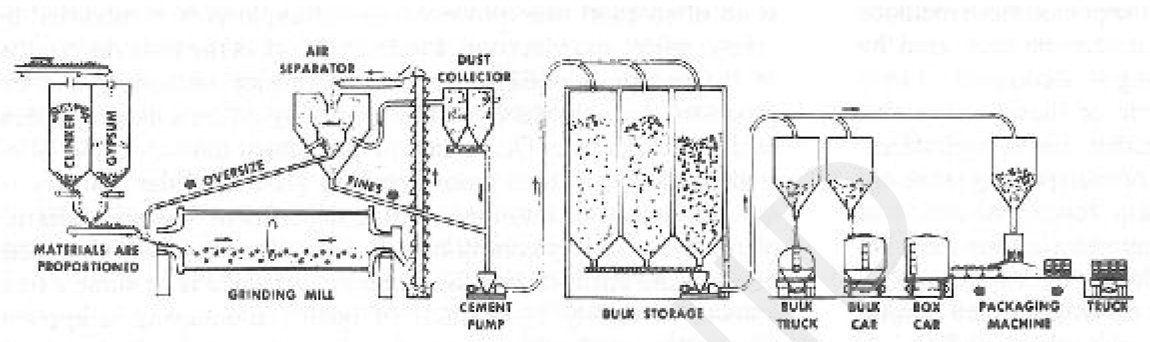


Fig. 3.5: Processing – finish grinding (Austin, 1984)

Majority of the cement grinding systems are closed circuit in which air separators provide classification. This is necessary because of marketplace strength requirements typically dictate that the cement be ground to not less than 3300 Blaine and more often 4000 Blaine. High early strength and specialty cements are ground to 5000 to 7000 Blaine. A major development in the 1980s has been the adoption of high efficiency classifiers pioneered by the Onoda Cement Co. (Japan) and the development of a pre-grinding clinker crusher. This device, known as a roll press, places the clinker under intense pressure between two counter-rotating rolls, causing fractures along grain boundaries. In the subsequent grinding in a ball mill, much higher throughputs are achieved for a given power input. The benefit of both of these developments is a 30 to 35% reduction in total grinding power of at the typical Blaine fineness of type I. The type I is the general type of cement, approved for a wide range of uses, mainly construction. The reduction is twice as much on high fineness cements such as type III (differs from type I based on the clinker quality). The type III cements are developed for high early strength uses.

Careful proportioning of clinker and gypsum feed to the mills is a requisite, and the additions of air-entraining agents or other small amounts of ingredients must be accurately metered.

The product is sampled and its quality verified, before it is loaded for shipment. Concrete storage silos are the major structures of the storage and shipping department, and that department is a significantly large capital cost factor of the whole plant. A feature of the U.S. and Canadian cement plants is that very small proportion of cement is shipped in bags. Several plants ship a majority of their output (either cement or clinker) by barge or by ship to terminals (silo type storage warehouses) and to satellite grinding plants. In Pakistan, major portion of the cement products is shipped and distributed to the consumers in bags.

CHAPTER 4: CEMENT QUARRY PLANNING

4.1. Introduction

A cement manufacturing operation may be receiving raw materials from a single or multiple quarries. However, limestone quarry providing high calcium raw materials is the prerequisite for a cement manufacturing operation. Some operations are fortunate enough to have clay, sandstone, slate, etc. reserves in the vicinity. Such operations may feed raw materials to the plant from multiple quarries. As focus of this research is the provision of raw materials at minimum cost subject to the satisfaction of quality and quantity constraints of the plant, therefore, this chapter discusses the planning issues related to the source providing these raw materials.

Quarry planning is a major engineering task in the development of a quarry. There are four groups of factors to be considered:

1. Natural and geologic factors, including geologic conditions, ore types, topography, and chemistry of limestone.
2. Economic factors, including waste and ore tonnage, operating and capital costs, production rate, market conditions.
3. Technological factors, including equipment, pit slope, bench height, etc.
4. Variability

These tasks are combined into long- and short-range quarry plans. Long term quarry planning and design tasks are typically in focus when designing the cement plant. These are not of the nature that one needs to repeat every day, week or month. These tasks are re-assessed with long term intervals, or if special circumstances develop. These circumstances can be; change in production, leasing and land acquisition issues, change in management and replacement of machinery etc. Contrary to long term planning, day to day computations are certainly relevant and imperative in short term planning for optimization and operational control tasks.

4.2. Long-range quarry planning

Typical steps in long-range quarry planning include:

1. Mapping of quarry chemistry from borehole data and the determination of long-range available resource and reserves.

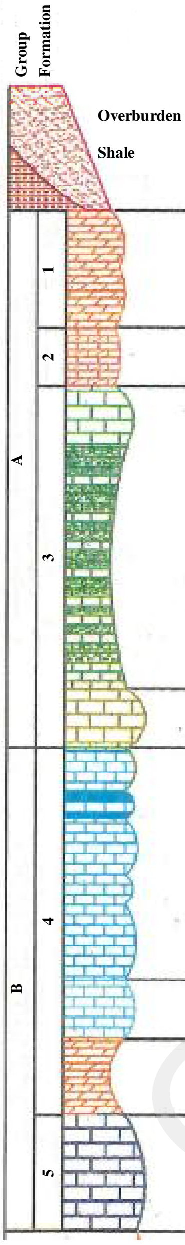
2. Assessment of quarry lifetime
3. Development of an operational plan based on the life of quarry production plans. The steps in operational plan include the following, however, it is not strictly limited to the these steps and varies from operation to operation:
    a. Selection of the most appropriate bench layouts i.e. height, width, face angle, etc.
    b. Analysis of alternative access routes to the reserves and selection of the most appropriate initial access based the minimum travelling distance for haulage equipment as the quarry develops laterally and vertically.
    c. Development of haulroad profile.
    d. Equipment selection.
4. Development of life of quarry production plan to establish the suitability of the reserves for provision of raw materials and requirement of additives to ensure sustained supply of raw mix to the plant.

Long-range planning begins with reserve estimation. The chemistry of limestone and the available tons are determined. At least thirty to fifty years of proven reserves are required for cement manufacturing; however, it depends upon the company’s policy. Two models are developed in this regard:

1. Geologic model
2. Chemical model

The geologic model identifies the rock types, overburden, the thickness of the particular limestone bed, etc. The geological column of the deposit helps sometimes in the determination of bench heights. The chemical model determines the chemical values of the different zones of the deposit. If the reserves meet chemical standards and if they have a sufficient volume of material, then the installation of a cement manufacturing plant becomes feasible, since this is one of the criteria. Fig. 4.1 presents a combined geologic and chemical model of a typical limestone reserve.

The development of a computerized quarry raw materials inventory in terms of a block model is the follow up of the geologic and chemical models. The aim of quarry block modeling is to provide the most accurate inventory of available raw materials.



GEOLOGICAL MEMBER	LITHOLOGY	ROCK TYPE	SiO_2 (%)	CaO (%)	MgO (%)	SO_3 (%)
a	DOLOMITE 0 - 25′	RT 1	13.0	27.5	13.0	1.06
b	DOLOMITIC LIMESTONE 0 - 10′	RT 2	7.7	42.3	4.8	0.74
c	HIGH CALCIUM LIMESTONE 0 - 8′	RT 3	3.2	52.8	0.85	0.18
	SHALEY LIMESTONE 25′ - 45′	RT 4	9.85	43.7	1.8	0.56
d	HIGH CALCIUM LIMESTONE 10′	RT 5	3.1	52.0	1.5	0.36
e	HIGH CALCIUM LIMESTONE 40′					
f	LIMESTONE 8.5′					
g	DOLOMITE 13′	RT 6	9.85	30.7	10.8	1.56
h	HIGH CALCIUM LIMESTONE 20′	RT 7	1.5	53.8	0.8	0.34

Fig. 4.1: Combined geologic and chemical model of a typical limestone deposit

When computing the model, the deposit is subdivided into a large number of small blocks, each representing a convenient mining quantity as a weekly or monthly production. From drillholes and other exploration data, some particular values are assigned to each block by means of interpolation techniques. In other words, a limited amount of information (drillholes, geology, geophysical results, etc.) is converted into the best possible continuous deposit description. The use of sophisticated computer methods vs. the use of traditional methods may lead to plus or minus 20 to 30 percent predictions in the error of estimate.

Using computer-aided techniques, uncertainties can be assessed at plus or minus five percent or even lower depending upon the degree of input. Furthermore, such uncertainties can be approximately defined, significantly reducing the risk of failure of project and resulting in higher productivity and cost savings (Baumgartner and Honerkamp, 1992).

In recent years, most mine plans have been based on block models. The block model generates an acceptable inventory of the deposit, describing for each location (block) the expected quality and quantity of material.

It is today's most powerful tool for raw materials evaluation and management. Based on the geologic information, either distance weighting techniques or geostatistical methods are used to estimate block values for the creation of the block model. Chemical values and material characteristics are assigned to each block. In a limestone quarry block model chemical values (percent content of SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, LOI, SO_3, K_2O, Na_2O, TiO_2, P_2O_5, and Cl) are assigned to each block. Fig. 4.2 shows the three-dimensional view of a block model and Table 4.1 presents the contents of a block model file for limestone quarries.

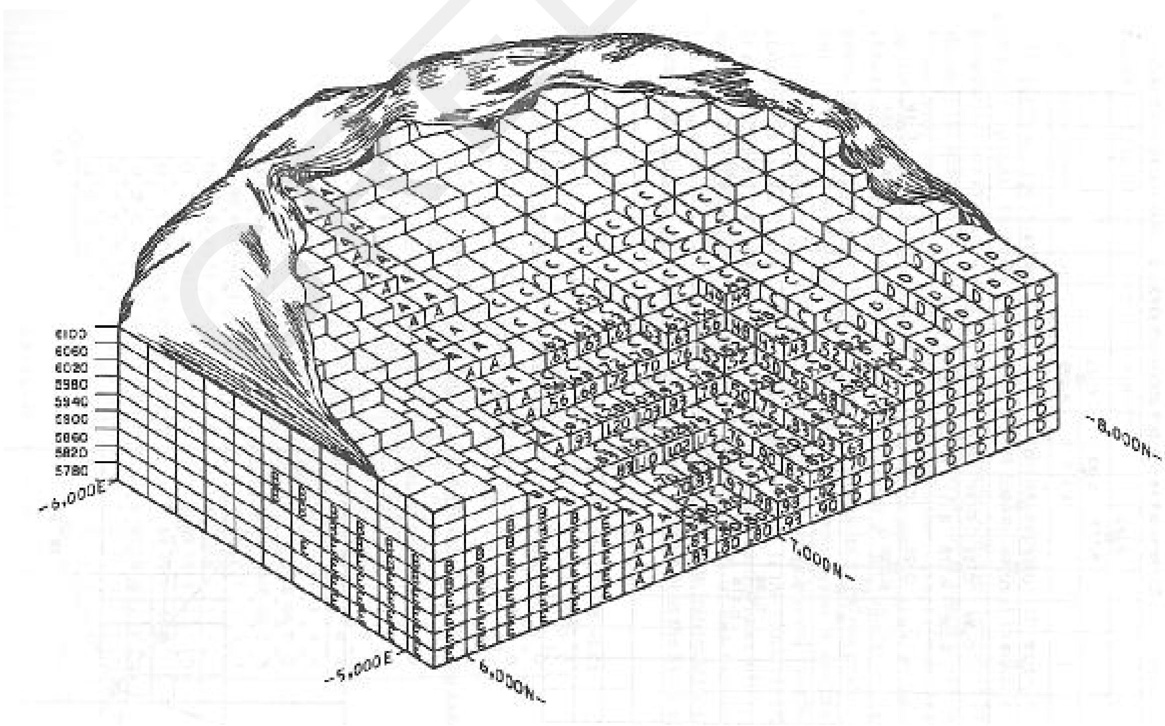


Fig. 4.2: Three-dimensional view of block model of a hypothetical deposit

Table 4.1: Information in the block model of typical limestone deposit

Item	Units
Block ID	NA
Rock Type	NA
Block Location (x, y, z coordinates)	NA
Block Size (x, y, z-direction)	Feet or Meters
Available Tons	Tons
SiO_2	%
Al_2O_3	%
Fe_2O_3	%
CaO	%
MgO	%
LOI	%
Others	%

The development of long-range i.e. life of quarry production plan of the cement quarry operations follows the definition of quarry life span and operational plan, because, outputs of the activities from the development of quarry block model to the quarry operational plan serve as an input to the quarry production planning models.

4.3. Short-range quarry planning

Once a long-range quarry plan has been established, a series of short-range quarry plans must be developed to implement the long-range plans. These define the intermediate steps required to ascertain final pit limits under physical, operating, and legal constraints. Following steps are common in short-range quarry planning (Rehman, 2007):

1. Data exchange between long term planning system and short term quality control system.
2. Dynamic estimation of quarry source material chemistry.
3. Raw mix optimization to meet current stockpile target chemistry.
    a. Grade control of the incoming material streams.
    b. Cost optimization to ensure lowest possible operating costs of feeding stockpiles.
4. Loading, haulage, and other equipment management.
5. Data storage, visualization and reporting.

Implementation of the company's policy to expose enough raw materials for sustained and uninterrupted supply to the plant also requires important overburden stripping decisions in short-range planning. They include whether or not to conduct advanced stripping, the amount of advance stripping, and stripping sequence.

Stripping sequence - how far in advance should certain zones of the deposit be stripped - is an operating variable, which must be investigated very carefully. An important element of short-range planning is the programming of stripping in such a way that it is not excessive during any period, especially at the outset of exploitation. The geometry of the deposit and the thickness of overburden determine the degree of advanced stripping. As a general rule pre-production stripping is kept to a minimum, so that raw materials can be fed to the plant as soon as possible.

Most cement manufacturing companies spend more shifts on the development work i.e. in the overburden removal so that the raw materials (limestone) mining is not handicapped by the overburden stripping. This approach provides enough open area for the mining on ore benches. Any overburden is usually stripped and transported to a disposal area in order to uncover the mineral deposit. However, in majority situations the overburden is rich in clay materials, i.e., silica, alumina, etc., which can also be used as raw mix in the plant, however, in such situations this is not considered as overburden as it is used as a resource.

It is worth mentioning that the development work consumes the quarry production capacity of the lower benches, as it requires sharing of the equipment from the lower to the upper benches. Similarly, equipment maintenance activity may lead to zero or minimum production for a particular bench. Chapter 7 presents sensitivity analysis with respect to economic impact of these frequent decisions as part of the short-range production planning.

As mentioned in chapter 1, this research focuses on very important aspect of the short-range planning i.e. the determination of amount of raw materials coming from various zones/portions or benches of the quarry and the market. However, the task will be accomplished such that the raw materials cost is minimized by fulfilling the quantity and quality requirements of the plant.

CHAPTER 5: RAW MATERIALS BLENDING MODEL

5.1. Blending requirement in cement manufacturing

Short-range production planning of raw materials in cement quarry operations is primarily concerned with developing a sequence of depletion schedules, beginning from the initial condition of the deposit to the final mine limits. However, there is an inherent task of preparation of raw mix from the run-of-mine material before cement production. The objective here is to mine in such a way that the resulting raw mix meets both quality and quantity specifications of the processing plant. The raw mixing problem is very critical in short-range planning, where the planner is concerned with reducing fluctuations in the chemistry of run-of-mine material, and with how much to mine in order to satisfy the tonnage and composition demands. Preparation of raw mix is beneficial for a number of reasons: it can improve the processing plant efficiency, and it minimizes the need for selective mining, hence reducing mining costs and increasing production.

Table 5.1 presents an average raw mix for cement manufacturing. The primary requirement for developing an acceptable raw mix is a source of calcium oxide (CaO), which is generally available from cement quarry as calcium carbonate ($CaCO_3$). Usually, a limestone quarry with an average CaO content of 48% is considered feasible for cement manufacturing. Secondary raw materials are required to achieve a balance of silica (SiO_2), alumina (Al_2O_3), and iron (Fe_2O_3). Silica ratio (SR), lime saturation factor (LSF) and alumina ratio (AM) are indices presented in equations (1), (2) and (3) which help in achieving the balance of main oxides (Rehman et. al, 2008):

$$SR = \frac{SiO_2}{Al_2O_3 + Fe_2O_3} \tag{1}$$

$$LSF = \frac{CaO}{2.8SiO_2 + 1.18Al_2O_3 + 0.65Fe_2O_3} \tag{2}$$

$$AM = \frac{Al_2O_3}{Fe_2O_3} \tag{3}$$

Raw mix and fuels introduced into the cement kiln for manufacturing of cement also contain some undesirable components. Such components, if present above the defined levels of concentration, may hinder the efficiency of manufacturing process. Magnesium compounds such as magnesia (MgO) are the most familiar of these. At low levels of concentration, the role of magnesia as fluxing agent is beneficial; however, as the concentration increases 3%, it becomes an impurity as it causes expansion/disruption of concrete. Similarly, the alkali content shall be kept at less than 1% for regular quality cement and less than 0.6% for low alkali cement. Higher alkali content leads to unsuitable deposits in cement kiln; hence, it becomes a hindrance in the smooth manufacturing process. Cement kiln stability also requires chlorine (Cl) concentration less than 0.05%.

Table 5.1: Raw materials percent content for cement manufacturing (Rehman et. al, 2008)

Chemical	**Percent Content**	
	Minimum	**Maximum**
SiO_2	14.00	15.00
Al_2O_3	2.70	3.40
Fe_2O_3	1.65	2.17
CaO	40.00	42.00
LOI	35.00	40.00
MgO	0.00	2.00
Na_2O	0.00	0.50
K_2O	0.00	0.50

Upon complete burning of the raw mix in cement kiln, a synthetic mineral mixture "clinker" is produced. The clinker consists of alite ($3CaO.SiO_2$) represented as "C_3S", belite ($2CaO.SiO_2$) represented as "C_2S", aluminate ($3CaO.Al_2O_3$) represented as "C_3A", and brownmillerite ($4CaO.Al_2O_3.Fe_2O_3$) represented as "C_4AF". Being the most critical constituents of clinker, percent content of C_3S, C_2S, C_3A, and C_4AF also provides balance of CaO, SiO_2, Al_2O_3, and Fe_2O_3 in raw mix. As given in equations (4) through (7), C_3S, C_2S, C_3A, and C_4AF shall range from 30% to 35%, 15% to 20%, 5% to 8%, 5% to 8%, respectively (Rehman et. al, 2008):

$$C_3S = 4.071 \times CaO - 7.6 \times SiO_2 - 6.718 \times Al_2O_3 - 1.43 \times Fe_2O_3 \quad (4)$$

$$C_2S = -3.071 \times CaO + 8.6 \times SiO_2 + 5.068 \times Al_2O_3 - 1.079 \times Fe_2O_3 \quad (5)$$

$$C_3A = 2.65 \times Al_2O_3 - 1.692 \times Fe_2O_3 \quad (6)$$

$$C_4AF = 3.043 \times Fe_2O_3 \quad (7)$$

5.2. Linear programming (LP) model

Optimum multi-period short-range production planning model for cement quarry operations attempts to develop a raw mix stockpile for cement plant, such that, this particular stockpile meets all quantity and quality requirements at the least possible cost. Therefore, the model develops a short-range production plan at the most micro level possible. Depending upon the kiln capacity, a typical cement plant consumes a raw mix stockpile in a couple of days. Following notations are defined for the indices, parameters, and variables of the LP model:

5.2.1. Indices

n: period index, where n = 1, …, n′

i: quarry zone/bench index, where i = 1, …, i′

j: additive (from market) index, where j = 1, …, j′
additives include shale, slate stone, clay, iron ore, and fly ash, etc.

k: chemical index, where k = 1, …, k′
chemicals include SiO_2, Al_2O_3, Fe_2O_3, CaO, C_3S, C_2S, C_3A, and C_4AF, etc.

5.2.2. Parameters

CQ_{in}: cost ($/ton) of raw materials from quarry zone/bench *i* during period *n*

CA_{jn}: cost ($/ton) of raw materials from additive *j* during period *n*

$QQMin_{in}$: minimum quantity (tons) of available raw materials (to be mined) from quarry zone *i* during period *n*

$QQMax_{in}$: maximum quantity (tons) of available raw materials (to be mined) from quarry zone *i* during period *n*

$QAMin_{jn}$: minimum quantity (tons) of additive *j* available during period *n*

$QAMax_{jn}$: maximum quantity (tons) of additive *j* available during period *n*

M_n: quarry raw materials mining capacity during period *n*

R_n: raw mix stockpile capacity during period *n*

ab_{kin}: available percent content of chemical *k* in raw material from quarry zone *i* during period *n*

aa_{kjn}: available percent content of chemical *k* in additive *j* during period *n*

l_{kn}: minimum percent content of a chemical *k* required in a given period *n*

u_{kn}: maximum percent content of chemical *k* required in a given period *n*

5.2.3. Variables

X_{in}: quantity (tons) of raw materials from quarry zone *i* during period *n*

Y_{jn}: quantity (tons) of raw materials from additive *j* during period *n*

5.2.4. Objective function

$$\text{Minimize cost} = \sum_{n=1}^{n'} \left[\sum_{i=1}^{i'} CQ_{in} X_{in} + \sum_{j=1}^{j'} CA_{jn} Y_{jn} \right] \qquad (8)$$

5.2.5. Constraints

5.2.5.1. Quantity of raw materials from quarry benches

Lower limit: $X_{in} \geq QQMin_{in} \quad \forall i, n$ (9)

Upper limit: $X_{in} \leq QQMax_{in} \quad \forall i, n$ (10)

5.2.5.2. Quantity of raw materials from additives

Lower limit: $Y_{jn} \geq QAMin_{jn} \quad \forall j, n$ (11)

Upper limit: $Y_{jn} \leq QAMax_{jn} \quad \forall j, n$ (12)

5.2.5.3. Raw materials mining capacity

$$\sum_{i=1}^{i'} X_{in} \leq M_n \quad \forall n \qquad (13)$$

5.2.5.4. Raw mix stockpile capacity

$$\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn} \geq R_n \quad \forall n \qquad (14)$$

5.2.5.5. Chemical content constraint

Lower limit:

$$\left[\sum_{i=1}^{i'} ab_{kin} X_{in} + \sum_{j=1}^{j'} aa_{kjn} Y_{jn} \right] - l_{kn} \left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn} \right] \geq 0.0 \quad \forall k, n \qquad (15)$$

Upper limit:

$$\left[\sum_{i=1}^{i'} ab_{kin} X_{in} + \sum_{j=1}^{j'} aa_{kjn} Y_{jn}\right] - u_{kn}\left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn}\right] \le 0.0 \quad \forall k, n \tag{16}$$

5.2.5.6. Silica ratio constraint

Lower limit:

$$\left[\begin{array}{l}\sum_{i=1}^{i'} ab_{(SiO_2)in} X_{in} + \\ \sum_{j=1}^{j'} aa_{(SiO_2)jn} Y_{jn}\end{array}\right] - l_{kn}\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in} + \sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right] + \\ \left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in} + \sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\end{array}\right] \ge 0.0 \ \forall n \tag{17}$$

Upper limit:

$$\left[\begin{array}{l}\sum_{i=1}^{i'} ab_{(SiO_2)in} X_{in} + \\ \sum_{j=1}^{j'} aa_{(SiO_2)jn} Y_{jn}\end{array}\right] - u_{kn}\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in} + \sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right] + \\ \left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in} + \sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\end{array}\right] \le 0.0 \quad \forall n \tag{18}$$

5.2.5.7. Lime saturation factor constraint

Lower limit:

$$\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(CaO)in} X_{in}\right] + \\ \left[\sum_{j=1}^{j'} aa_{(CaO)jn} Y_{jn}\right]\end{array}\right] - l_{kn}\left[\begin{array}{l}2.80 \times \left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(SiO_2)jn} Y_{jn}\right]\right] + \\ 1.18 \times \left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\right] + \\ 0.65 \times \left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right]\end{array}\right] \ge 0.0 \ \forall n \tag{19}$$

Upper limit:

$$\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(CaO)in} X_{in}\right] + \\ \left[\sum_{j=1}^{j'} aa_{(CaO)jn} Y_{jn}\right]\end{array}\right] - u_{kn}\left[\begin{array}{l}2.80 \times \left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(SiO_2)jn} Y_{jn}\right]\right] + \\ 1.18 \times \left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\right] + \\ 0.65 \times \left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right] + \left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right]\end{array}\right] \le 0.0 \ \forall n \tag{20}$$

5.2.5.8. Alumina ratio constraint

Lower limit:

$$\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right]+ \\ \left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\end{array}\right] - l_{kn}\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right] \geq 0.0 \;\; \forall n \qquad (21)$$

Upper limit:

$$\left[\begin{array}{l}\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right]+ \\ \left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\end{array}\right] - u_{kn}\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right] \leq 0.0 \;\; \forall n \qquad (22)$$

5.2.5.9. C_3S constraint

Lower limit:

$$\left[\begin{array}{l}4.071\times\left[\left[\sum_{i=1}^{i'} ab_{(CaO)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(CaO)jn} Y_{jn}\right]\right]- \\ 7.600\times\left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(SiO_2)jn} Y_{jn}\right]\right]- \\ 6.718\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\right]- \\ 1.430\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right]\end{array}\right] - l_{kn}\left[\left[\sum_{i=1}^{i'} X_{in}\right]+\left[\sum_{j=1}^{j'} Y_{jn}\right]\right] \geq 0.0 \;\; \forall n \qquad (23)$$

Upper limit:

$$\begin{bmatrix} 4.071\times\left[\left[\sum_{i=1}^{i'} ab_{(CaO)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(CaO)jn}Y_{jn}\right]\right]- \\ 7.600\times\left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(SiO_2)jn}Y_{jn}\right]\right]- \\ 6.718\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn}Y_{jn}\right]\right]- \\ 1.430\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn}Y_{jn}\right]\right] \end{bmatrix} - u_{kn}\left[\left[\sum_{i=1}^{i'} X_{in}\right]+\left[\sum_{j=1}^{j'} Y_{jn}\right]\right] \le 0.0 \;\forall n \quad (24)$$

5.2.5.10. C_2S constraint

Lower limit:

$$\begin{bmatrix} -3.071\times\left[\left[\sum_{i=1}^{i'} ab_{(CaO)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(CaO)jn}Y_{jn}\right]\right]+ \\ 8.600\times\left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(SiO_2)jn}Y_{jn}\right]\right]+ \\ 5.068\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn}Y_{jn}\right]\right]+ \\ 1.079\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn}Y_{jn}\right]\right] \end{bmatrix} - l_{kn}\left[\left[\sum_{i=1}^{i'} X_{in}\right]+\left[\sum_{j=1}^{j'} Y_{jn}\right]\right] \ge 0.0 \;\forall n \quad (25)$$

Upper limit:

$$\begin{bmatrix} -3.071\times\left[\left[\sum_{i=1}^{i'} ab_{(CaO)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(CaO)jn}Y_{jn}\right]\right]+ \\ 8.600\times\left[\left[\sum_{i=1}^{i'} ab_{(SiO_2)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(SiO_2)jn}Y_{jn}\right]\right]+ \\ 5.068\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn}Y_{jn}\right]\right]+ \\ 1.079\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in}X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn}Y_{jn}\right]\right] \end{bmatrix} - u_{kn}\left[\left[\sum_{i=1}^{i'} X_{in}\right]+\left[\sum_{j=1}^{j'} Y_{jn}\right]\right] \le 0.0 \;\forall n \quad (26)$$

5.2.5.11. C_3A constraint

Lower limit:

$$\left[\begin{array}{l} 2.650\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\right]- \\ 1.692\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right] \end{array}\right] - l_{kn}\left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn}\right] \geq 0.0 \quad \forall n \qquad (27)$$

Upper limit:

$$\left[\begin{array}{l} 2.650\times\left[\left[\sum_{i=1}^{i'} ab_{(Al_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Al_2O_3)jn} Y_{jn}\right]\right]- \\ 1.692\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right] \end{array}\right] - u_{kn}\left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn}\right] \leq 0.0 \quad \forall n \qquad (28)$$

5.2.5.12. C_4AF constraint

Lower limit:

$$\left[3.043\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right]\right] - l_{kn}\left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn}\right] \geq 0.0 \quad \forall n \qquad (29)$$

Upper limit:

$$\left[3.043\times\left[\left[\sum_{i=1}^{i'} ab_{(Fe_2O_3)in} X_{in}\right]+\left[\sum_{j=1}^{j'} aa_{(Fe_2O_3)jn} Y_{jn}\right]\right]\right] - u_{kn}\left[\sum_{i=1}^{i'} X_{in} + \sum_{j=1}^{j'} Y_{jn}\right] \leq 0.0 \quad \forall n \qquad (30)$$

CHAPTER 6: APPLICATION OF THE LP MODEL

6.1. LP model – input data

This chapter presents an application of the blending optimization model for the development of two-period short range production plan using data from an existing cement manufacturing operation in northern Pakistan.

The operation utilizes a minimum of 15000 tons of raw materials for producing cement. Therefore, honoring the minimum raw mix stockpile capacity of 15,000 tons, the LP model generates an optimum short-range production plan for this particular application. It is worth mentioning that this is the most micro level raw materials production plan that may be developed for a particular operation.

The limestone quarry operation consists of five benches/zones. Table 6.1 presents the cost (Rs./ton) of mining (drill + blast + load + haul) raw materials from a particular area (either ready for blast or already blasted) of the bench along with the quantity (tons) of available raw materials on each bench. The change in cost of mining from bench 1 to 5 is dependent upon the change in haulage or raw materials transportation cost. As this particular cost may increase or decrease depending upon the haulage distance between loading site on a particular bench and the crusher.

Table 6.1: Cost and quantity of the raw materials from limestone quarry

Bench	Cost (Rs./ton)		Quantity (tons)			
	Period 1	Period 2	Period 1		Period 2	
			Minimum	Maximum	Minimum	Maximum
1	20.50	18.00	1000	2500	2000	3000
2	23.00	20.30	3000	4500	2500	3500
3	25.50	26.00	3000	4000	4000	5000
4	27.20	25.30	1000	2500	1500	2500
5	27.50	28.30	1000	2500	1000	2500

Table 6.2 shows the purchasing cost (Rs./ton) and the quantity (tons) of available additives including clay, slate stone, shale, and laterite.

Table 6.2: Cost and quantity of the additives

Additive	Cost (Rs./ton)		Quantity (tons)			
	Period 1	Period 2	Period 1		Period 2	
			Minimum	Maximum	Minimum	Maximum
Clay	16.40	16.40	1500	3000	2000	4000
Slate Stone	70.60	70.60	400	1100	400	1500
Shale	18.80	18.80	0	500	0	500
Laterite	352.50	352.50	0	150	0	150

Table 6.3 and 6.4 present the chemistry of a particular portion/block/area of the bench, which is either ready for the next blast or may have been blasted earlier during period 1 or 2, respectively. During short-range production planning stage the exact chemistry of an area is known from the chemical analysis of the samples recovered from the blast (drill) holes.

Table 6.3: Percent chemical content of the five benches/zones of the limestone quarry during period 1

Chemical (%)	Bench 1	Bench 2	Bench 3	Bench 4	Bench 5
CaO	48.50	46.50	49.50	50.05	44.01
SiO_2	3.25	5.01	1.75	2.5	9.04
Al_2O_3	1.83	0.99	1.50	0.95	1.99
Fe_2O_3	1.12	1.25	0.20	1.05	1.5
LOI	43.75	43.89	44.44	41.91	42.38
MgO	0.99	1.80	2.05	2.99	0.50
Na_2O	0.27	0.25	0.24	0.30	0.27
K_2O	0.29	0.31	0.32	0.25	0.31

Table 6.4: Percent chemical content of the five benches/zones of the limestone quarry during period 2

Chemical (%)	Bench 1	Bench 2	Bench 3	Bench 4	Bench 5
CaO	47.20	45.30	51.25	52.08	41.83
SiO_2	2.95	5.21	1.63	2.69	7.56
Al_2O_3	1.96	0.93	1.42	0.91	1.86
Fe_2O_3	1.15	1.23	0.19	0.96	1.65
LOI	45.44	44.92	43.12	40.11	46.00
MgO	0.93	1.78	1.96	2.64	0.45
Na_2O	0.14	0.28	0.17	0.39	0.29
K_2O	0.23	0.35	0.26	0.22	0.36

The chemistry of the additives including clay, slate, shale, and laterite (iron ore) is presented in Tables 6.5 and 6.6 for each of periods 1 and 2, respectively.

Table 6.5: Percent chemical content of the additives during period 1

Chemical (%)	Clay	Slate	Shale	Laterite
CaO	11.27	0.72	2.99	1.69
SiO_2	51.51	76.69	58.28	32.38
Al_2O_3	12.14	10.17	13.92	13.07
Fe_2O_3	5.35	5.74	6.53	34.09
LOI	16.75	5.08	16.13	16.90
MgO	2.50	1.06	1.62	1.32
Na_2O	0.27	0.32	0.29	0.31
K_2O	0.21	0.22	0.24	0.24

Table 6.6: Percent chemical content of the additives during period 2

Chemical (%)	Clay	Slate	Shale	Laterite
CaO	10.18	0.79	2.63	1.98
SiO_2	52.00	75.30	59.13	29.46
Al_2O_3	11.89	10.03	13.25	12.69
Fe_2O_3	5.15	5.06	6.25	36.24
LOI	18.00	7.32	16.88	18.23
MgO	2.36	0.97	1.54	0.84
Na_2O	0.23	0.29	0.15	0.34
K_2O	0.19	0.24	0.17	0.22

6.2. LP formulation

The linear programming formulation for the development of a two-period short range production plan is presented as follows:

6.2.1. Objective function

$$\text{Minimize cost} = \begin{array}{l} 20.50X_{11} + 23.00X_{21} + 25.50X_{31} + 27.20X_{41} + 27.50X_{51} + \\ 16.40Y_{11} + 70.60Y_{21} + 18.80Y_{31} + 352.50Y_{41} + \\ 18.00X_{12} + 20.30X_{22} + 26.00X_{32} + 25.30X_{42} + 28.30X_{52} + \\ 16.40Y_{12} + 70.60Y_{22} + 18.80Y_{32} + 352.50Y_{42} \end{array}$$

6.2.2. Quantity of raw materials (quarry benches) constraint

Lower limit:

For $n = 1$, $X_{11} \geq 1000, X_{21} \geq 3000, X_{31} \geq 3000, X_{41} \geq 1000, X_{51} \geq 1000$

For $n = 2$, $X_{12} \geq 2000, X_{22} \geq 2500, X_{32} \geq 4000, X_{42} \geq 1500, X_{52} \geq 1000$

Upper limit:

For $n = 1$, $X_{11} \leq 2500, X_{21} \leq 4500, X_{31} \leq 4000, X_{41} \leq 2500, X_{51} \leq 2500$

For $n = 2$, $X_{12} \leq 3000, X_{22} \leq 3500, X_{32} \leq 5000, X_{42} \leq 2500, X_{52} \leq 2500$

6.2.3. Quantity of raw materials (additives) constraint

Lower limit:

For $n = 1$, $Y_{11} \geq 1500, Y_{21} \geq 400, Y_{31} \geq 0, Y_{41} \geq 0$

For $n = 2$, $Y_{12} \geq 2000, Y_{22} \geq 400, Y_{32} \geq 0, Y_{42} \geq 0$

Upper limit:

For $n = 1$, $Y_{11} \leq 3000, Y_{21} \leq 1100, Y_{31} \leq 500, Y_{41} \leq 150$

For $n = 2$, $Y_{12} \leq 4000, Y_{22} \leq 1500, Y_{32} \leq 500, Y_{42} \leq 150$

6.2.4. Raw materials mining capacity constraint

For $n = 1$, $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \leq 15000$

For $n = 2$, $X_{12} + X_{22} + X_{32} + X_{42} + X_{52} \leq 15000$

6.2.5. Raw mix stockpile capacity constraint

For $n = 1$, $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + Y_{11} + Y_{21} + Y_{31} + Y_{41} \geq 15000$

For $n = 2$, $X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + Y_{12} + Y_{22} + Y_{32} + Y_{42} \geq 15000$

6.2.6. Chemical content constraint

Lower limit (CaO):

For $n = 1$,
$$\left[\begin{bmatrix} 48.50X_{11} + 46.50X_{21} + 49.50X_{31} + 50.05X_{41} + 44.01X_{51} + \\ 11.27Y_{11} + 0.72Y_{21} + 2.99X_{31} + 1.69Y_{41} \end{bmatrix} - \atop 40.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.0$$

For n = 2, $$\left[\begin{bmatrix} 47.20X_{12} + 45.30X_{22} + 51.25X_{32} + 52.08X_{42} + 41.83X_{52} + \\ 10.18Y_{12} + 0.79Y_{22} + 2.63X_{32} + 1.98Y_{42} \end{bmatrix} - 40.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.0$$

Upper limit (CaO):

For n = 1, $$\left[\begin{bmatrix} 48.50X_{11} + 46.50X_{21} + 49.50X_{31} + 50.05X_{41} + 44.01X_{51} + \\ 11.27Y_{11} + 0.72Y_{21} + 2.99X_{31} + 1.69Y_{41} \end{bmatrix} - 42.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.0$$

For n = 2, $$\left[\begin{bmatrix} 47.20X_{12} + 45.30X_{22} + 51.25X_{32} + 52.08X_{42} + 41.83X_{52} + \\ 10.18Y_{12} + 0.79Y_{22} + 2.63X_{32} + 1.98Y_{42} \end{bmatrix} - 42.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.0$$

Lower limit (SiO_2):

For n = 1, $$\left[\begin{bmatrix} 3.25X_{11} + 5.01X_{21} + 1.75X_{31} + 2.50X_{41} + 9.04X_{51} + \\ 51.51Y_{11} + 76.69Y_{21} + 58.28X_{31} + 32.38Y_{41} \end{bmatrix} - 14.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.0$$

For n = 2, $$\left[\begin{bmatrix} 2.95X_{12} + 5.21X_{22} + 1.63X_{32} + 2.69X_{42} + 7.56X_{52} + \\ 52.00Y_{12} + 75.30Y_{22} + 59.13X_{32} + 29.46Y_{42} \end{bmatrix} - 14.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.0$$

Upper limit (SiO_2):

For n = 1, $$\left[\begin{bmatrix} 3.25X_{11} + 5.01X_{21} + 1.75X_{31} + 2.50X_{41} + 9.04X_{51} + \\ 51.51Y_{11} + 76.69Y_{21} + 58.28X_{31} + 32.38Y_{41} \end{bmatrix} - 15.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.0$$

For n = 2, $$\left[\begin{bmatrix} 2.95X_{12} + 5.21X_{22} + 1.63X_{32} + 2.69X_{42} + 7.56X_{52} + \\ 52.00Y_{12} + 75.30Y_{22} + 59.13X_{32} + 29.46Y_{42} \end{bmatrix} - 15.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.00$$

Lower limit (Al_2O_3):

$$\text{For } n = 1,\ \left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} - 2.70\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.0$$

$$\text{For } n = 2,\ \left[\begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} - 2.70\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.0$$

Upper limit (Al_2O_3):

$$\text{For } n = 1,\ \left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} - 3.40\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.0$$

$$\text{For } n = 2,\ \left[\begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} - 3.40\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.0$$

Lower limit (Fe_2O_3):

$$\text{For } n = 1,\ \left[\begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix} - 1.65\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.0$$

$$\text{For } n = 2,\ \left[\begin{bmatrix} 1.15X_{12} + 1.23X_{22} + 0.19X_{32} + 0.96X_{42} + 1.65X_{52} + \\ 5.15Y_{12} + 5.06Y_{22} + 6.25X_{32} + 36.24Y_{42} \end{bmatrix} - 1.65\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.0$$

Upper limit (Fe_2O_3):

$$\text{For } n = 1,\ \left[\begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix} - 2.17\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.0$$

For n = 2,
$$\left[\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}-\\ 2.17\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\leq 0.0$$

Lower limit (LOI):

For n = 1,
$$\left[\begin{bmatrix}43.75X_{11}+43.89X_{21}+44.44X_{31}+41.91X_{41}+42.38X_{51}+\\ 16.75Y_{11}+5.08Y_{21}+16.13X_{31}+16.90Y_{41}\end{bmatrix}-\\ 35.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\ Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\geq 0.0$$

For n = 2,
$$\left[\begin{bmatrix}45.44X_{12}+44.92X_{22}+43.12X_{32}+40.11X_{42}+46.00X_{52}+\\ 18.00Y_{12}+7.32Y_{22}+16.88X_{32}+18.23Y_{42}\end{bmatrix}-\\ 35.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\geq 0.0$$

Upper limit (LOI):

For n = 1,
$$\left[\begin{bmatrix}43.75X_{11}+43.89X_{21}+44.44X_{31}+41.91X_{41}+42.38X_{51}+\\ 16.75Y_{11}+5.08Y_{21}+16.13X_{31}+16.90Y_{41}\end{bmatrix}-\\ 40.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\ Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\leq 0.0$$

For n = 2,
$$\left[\begin{bmatrix}45.44X_{12}+44.92X_{22}+43.12X_{32}+40.11X_{42}+46.00X_{52}+\\ 18.00Y_{12}+7.32Y_{22}+16.88X_{32}+18.23Y_{42}\end{bmatrix}-\\ 40.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\leq 0.0$$

Lower limit (MgO):

For n = 1,
$$\left[\begin{bmatrix}0.99X_{11}+1.80X_{21}+2.05X_{31}+2.99X_{41}+0.50X_{51}+\\ 2.50Y_{11}+1.06Y_{21}+1.62X_{31}+1.32Y_{41}\end{bmatrix}-\\ 0.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\ Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\geq 0.00$$

For n = 2,
$$\left[\begin{bmatrix}0.93X_{12}+1.78X_{22}+1.96X_{32}+2.64X_{42}+0.45X_{52}+\\ 2.36Y_{12}+0.97Y_{22}+1.54X_{32}+0.84Y_{42}\end{bmatrix}-\\ 0.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\geq 0.00$$

Upper limit (MgO):

For $n = 1$, $$\left[\begin{bmatrix} 0.99X_{11} + 1.80X_{21} + 2.05X_{31} + 2.99X_{41} + 0.50X_{51} + \\ 2.50Y_{11} + 1.06Y_{21} + 1.62X_{31} + 1.32Y_{41} \end{bmatrix} - 2.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.00$$

For $n = 2$, $$\left[\begin{bmatrix} 0.93X_{12} + 1.78X_{22} + 1.96X_{32} + 2.64X_{42} + 0.45X_{52} + \\ 2.36Y_{12} + 0.97Y_{22} + 1.54X_{32} + 0.84Y_{42} \end{bmatrix} - 2.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.00$$

Lower limit (Na_2O):

For $n = 1$, $$\left[\begin{bmatrix} 0.27X_{11} + 0.25X_{21} + 0.24X_{31} + 0.30X_{41} + 0.27X_{51} + \\ 0.27Y_{11} + 0.32Y_{21} + 0.29X_{31} + 0.31Y_{41} \end{bmatrix} - 0.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.00$$

For $n = 2$, $$\left[\begin{bmatrix} 0.14X_{12} + 0.28X_{22} + 0.17X_{32} + 0.39X_{42} + 0.29X_{52} + \\ 0.23Y_{12} + 0.29Y_{22} + 0.15X_{32} + 0.34Y_{42} \end{bmatrix} - 0.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.00$$

Upper limit (Na_2O):

For $n = 1$, $$\left[\begin{bmatrix} 0.27X_{11} + 0.25X_{21} + 0.24X_{31} + 0.30X_{41} + 0.27X_{51} + \\ 0.27Y_{11} + 0.32Y_{21} + 0.29X_{31} + 0.31Y_{41} \end{bmatrix} - 0.50\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.00$$

For $n = 2$, $$\left[\begin{bmatrix} 0.14X_{12} + 0.28X_{22} + 0.17X_{32} + 0.39X_{42} + 0.29X_{52} + \\ 0.23Y_{12} + 0.29Y_{22} + 0.15X_{32} + 0.34Y_{42} \end{bmatrix} - 0.50\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.00$$

Lower limit (K_2O):

For $n = 1$, $$\left[\begin{bmatrix} 0.29X_{11} + 0.31X_{21} + 0.32X_{31} + 0.25X_{41} + 0.31X_{51} + \\ 0.21Y_{11} + 0.22Y_{21} + 0.24X_{31} + 0.24Y_{41} \end{bmatrix} - 0.00\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \geq 0.00$$

For n = 2,
$$\left[\begin{bmatrix} 0.23X_{12} + 0.35X_{22} + 0.26X_{32} + 0.22X_{42} + 0.36X_{52} + \\ 0.19Y_{12} + 0.24Y_{22} + 0.17X_{32} + 0.22Y_{42} \end{bmatrix} - 0.00\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \geq 0.00$$

Upper limit (K_2O):

For n = 1,
$$\left[\begin{bmatrix} 0.29X_{11} + 0.31X_{21} + 0.32X_{31} + 0.25X_{41} + 0.31X_{51} + \\ 0.21Y_{11} + 0.22Y_{21} + 0.24X_{31} + 0.24Y_{41} \end{bmatrix} - 0.50\begin{bmatrix} X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + \\ Y_{11} + Y_{21} + X_{31} + Y_{41} \end{bmatrix}\right] \leq 0.00$$

For n = 2,
$$\left[\begin{bmatrix} 0.23X_{12} + 0.35X_{22} + 0.26X_{32} + 0.22X_{42} + 0.36X_{52} + \\ 0.19Y_{12} + 0.24Y_{22} + 0.17X_{32} + 0.22Y_{42} \end{bmatrix} - 0.50\begin{bmatrix} X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + \\ Y_{12} + Y_{22} + X_{32} + Y_{42} \end{bmatrix}\right] \leq 0.00$$

6.2.7. Silica ratio constraint

Lower limit:

For n = 1,
$$\left[\begin{bmatrix} 3.25X_{11} + 5.01X_{21} + 1.75X_{31} + 2.50X_{41} + 9.04X_{51} + \\ 51.51Y_{11} + 76.69Y_{21} + 58.28X_{31} + 32.38Y_{41} \end{bmatrix} - 2.60\left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} + \begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix}\right]\right] \geq 0.0$$

For n = 2,
$$\left[\begin{bmatrix} 2.95X_{12} + 5.21X_{22} + 1.63X_{32} + 2.69X_{42} + 7.56X_{52} + \\ 52.00Y_{12} + 75.30Y_{22} + 59.13X_{32} + 29.46Y_{42} \end{bmatrix} - 2.60\left[\begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} + \begin{bmatrix} 1.15X_{12} + 1.23X_{22} + 0.19X_{32} + 0.96X_{42} + 1.65X_{52} + \\ 5.15Y_{12} + 5.06Y_{22} + 6.25X_{32} + 36.24Y_{42} \end{bmatrix}\right]\right] \geq 0.0$$

Upper limit:

For n = 1,
$$\left[\begin{bmatrix} 3.25X_{11} + 5.01X_{21} + 1.75X_{31} + 2.50X_{41} + 9.04X_{51} + \\ 51.51Y_{11} + 76.69Y_{21} + 58.28X_{31} + 32.38Y_{41} \end{bmatrix} - 2.90\left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} + \begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix}\right]\right] \leq 0.0$$

For $n = 2$,
$$\left[\begin{bmatrix}2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+\\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42}\end{bmatrix}- 2.90\left[\begin{bmatrix}1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+\\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42}\end{bmatrix}+\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\right]\right]\le 0.0$$

6.2.8. Lime saturation factor constraint

Lower limit:

For $n = 1$,
$$\left[\begin{bmatrix}48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+\\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41}\end{bmatrix}- 0.845\left[2.80\times\begin{bmatrix}3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+\\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41}\end{bmatrix}+1.18\times\begin{bmatrix}1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+\\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41}\end{bmatrix}+0.65\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\right]\right]\ge 0.0$$

For $n = 2$,
$$\left[\begin{bmatrix}47.20X_{12}+45.30X_{22}+51.25X_{32}+52.08X_{42}+41.83X_{52}+\\ 10.18Y_{12}+0.79Y_{22}+2.63X_{32}+1.98Y_{42}\end{bmatrix}- 0.845\left[2.80\times\begin{bmatrix}2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+\\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42}\end{bmatrix}+1.18\times\begin{bmatrix}1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+\\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42}\end{bmatrix}+0.65\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\right]\right]\ge 0.0$$

Upper limit:

For $n = 1$,
$$\left[\begin{bmatrix}48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+\\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41}\end{bmatrix}- 0.900\left[2.80\times\begin{bmatrix}3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+\\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41}\end{bmatrix}+1.18\times\begin{bmatrix}1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+\\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41}\end{bmatrix}+0.65\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\right]\right]\le 0.0$$

$$\text{For } n = 2, \quad \left[\begin{bmatrix} 47.20X_{12} + 45.30X_{22} + 51.25X_{32} + 52.08X_{42} + 41.83X_{52} + \\ 10.18Y_{12} + 0.79Y_{22} + 2.63X_{32} + 1.98Y_{42} \end{bmatrix} - \\ 0.900 \left[\begin{matrix} 2.80 \times \begin{bmatrix} 2.95X_{12} + 5.21X_{22} + 1.63X_{32} + 2.69X_{42} + 7.56X_{52} + \\ 52.00Y_{12} + 75.30Y_{22} + 59.13X_{32} + 29.46Y_{42} \end{bmatrix} \\ 1.18 \times \begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} + \\ 0.65 \times \begin{bmatrix} 1.15X_{12} + 1.23X_{22} + 0.19X_{32} + 0.96X_{42} + 1.65X_{52} + \\ 5.15Y_{12} + 5.06Y_{22} + 6.25X_{32} + 36.24Y_{42} \end{bmatrix} \end{matrix} \right] \right] \le 0.0$$

6.2.9. Alumina ratio constraint

Lower limit:

$$\text{For } n = 1, \quad \left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} - \\ 1.50 \begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix} \right] \ge 0.0$$

$$\text{For } n = 2, \quad \left[\begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} - \\ 1.50 \begin{bmatrix} 1.15X_{12} + 1.23X_{22} + 0.19X_{32} + 0.96X_{42} + 1.65X_{52} + \\ 5.15Y_{12} + 5.06Y_{22} + 6.25X_{32} + 36.24Y_{42} \end{bmatrix} \right] \ge 0.0$$

Upper limit:

$$\text{For } n = 1, \quad \left[\begin{bmatrix} 1.83X_{11} + 0.99X_{21} + 1.50X_{31} + 0.95X_{41} + 1.99X_{51} + \\ 12.14Y_{11} + 10.17Y_{21} + 13.92X_{31} + 13.07Y_{41} \end{bmatrix} - \\ 2.00 \begin{bmatrix} 1.12X_{11} + 1.25X_{21} + 0.20X_{31} + 1.05X_{41} + 1.50X_{51} + \\ 5.35Y_{11} + 5.74Y_{21} + 6.53X_{31} + 34.09Y_{41} \end{bmatrix} \right] \le 0.0$$

$$\text{For } n = 2, \quad \left[\begin{bmatrix} 1.96X_{12} + 0.93X_{22} + 1.42X_{32} + 0.91X_{42} + 1.86X_{52} + \\ 11.89Y_{12} + 10.03Y_{22} + 13.25X_{32} + 12.69Y_{42} \end{bmatrix} - \\ 2.00 \begin{bmatrix} 1.15X_{12} + 1.23X_{22} + 0.19X_{32} + 0.96X_{42} + 1.65X_{52} + \\ 5.15Y_{12} + 5.06Y_{22} + 6.25X_{32} + 36.24Y_{42} \end{bmatrix} \right] \le 0.0$$

6.2.10. C_3S constraint

Lower limit:

$$\text{For } n = 1,\quad \left[\begin{bmatrix} 4.071\times\begin{bmatrix} 48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+ \\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41} \end{bmatrix} - \\ 7.600\times\begin{bmatrix} 3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+ \\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41} \end{bmatrix} - \\ 6.718\times\begin{bmatrix} 1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+ \\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41} \end{bmatrix} - \\ 1.430\times\begin{bmatrix} 1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+ \\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41} \end{bmatrix} \end{bmatrix} - \\ 30.00\begin{bmatrix} X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+ \\ Y_{11}+Y_{21}+X_{31}+Y_{41} \end{bmatrix} \right] \geq 0.0$$

$$\text{For } n = 2,\quad \left[\begin{bmatrix} 4.071\times\begin{bmatrix} 47.20X_{12}+45.30X_{22}+51.25X_{32}+52.08X_{42}+41.83X_{52}+ \\ 10.18Y_{12}+0.79Y_{22}+2.63X_{32}+1.98Y_{42} \end{bmatrix} - \\ 7.600\times\begin{bmatrix} 2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+ \\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42} \end{bmatrix} - \\ 6.718\times\begin{bmatrix} 1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+ \\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42} \end{bmatrix} - \\ 1.43\times\begin{bmatrix} 1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+ \\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42} \end{bmatrix} \end{bmatrix} - \\ 30.00\begin{bmatrix} X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+ \\ Y_{12}+Y_{22}+X_{32}+Y_{42} \end{bmatrix} \right] \geq 0.0$$

Upper limit:

$$\text{For } n = 1,\quad \left[\begin{bmatrix} 4.071\times\begin{bmatrix} 48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+ \\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41} \end{bmatrix} - \\ 7.600\times\begin{bmatrix} 3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+ \\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41} \end{bmatrix} - \\ 6.718\times\begin{bmatrix} 1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+ \\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41} \end{bmatrix} - \\ 1.430\times\begin{bmatrix} 1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+ \\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41} \end{bmatrix} \end{bmatrix} - \\ 35.00\begin{bmatrix} X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+ \\ Y_{11}+Y_{21}+X_{31}+Y_{41} \end{bmatrix} \right] \leq 0.0$$

$$\text{For } n = 2,\quad \left[\begin{array}{l} \left[\begin{array}{l} 4.071\times\left[\begin{array}{c} 47.20X_{12}+45.30X_{22}+51.25X_{32}+52.08X_{42}+41.83X_{52}+ \\ 10.18Y_{12}+0.79Y_{22}+2.63X_{32}+1.98Y_{42} \end{array}\right]- \\ 7.600\times\left[\begin{array}{c} 2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+ \\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42} \end{array}\right]- \\ 6.718\times\left[\begin{array}{c} 1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+ \\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42} \end{array}\right]- \\ 1.43\times\left[\begin{array}{c} 1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+ \\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42} \end{array}\right] \end{array}\right]- \\ 35.00\left[\begin{array}{c} X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+ \\ Y_{12}+Y_{22}+X_{32}+Y_{42} \end{array}\right] \end{array}\right] \le 0.0$$

6.2.11. C_2S constraint

Lower limit:

$$\text{For } n = 1,\quad \left[\begin{array}{l} \left[\begin{array}{l} -3.071\times\left[\begin{array}{c} 48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+ \\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41} \end{array}\right]+ \\ 8.600\times\left[\begin{array}{c} 3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+ \\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41} \end{array}\right]+ \\ 5.068\times\left[\begin{array}{c} 1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+ \\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41} \end{array}\right]+ \\ 1.079\times\left[\begin{array}{c} 1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+ \\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41} \end{array}\right] \end{array}\right]- \\ 15.00\left[\begin{array}{c} X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+ \\ Y_{11}+Y_{21}+X_{31}+Y_{41} \end{array}\right] \end{array}\right] \ge 0.0$$

$$\text{For } n = 2,\quad \left[\begin{array}{l} \left[\begin{array}{l} -3.071\times\left[\begin{array}{c} 47.20X_{12}+45.30X_{22}+51.25X_{32}+52.08X_{42}+41.83X_{52}+ \\ 10.18Y_{12}+0.79Y_{22}+2.63X_{32}+1.98Y_{42} \end{array}\right]+ \\ 8.600\times\left[\begin{array}{c} 2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+ \\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42} \end{array}\right]+ \\ 5.068\times\left[\begin{array}{c} 1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+ \\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42} \end{array}\right]+ \\ 1.079\times\left[\begin{array}{c} 1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+ \\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42} \end{array}\right] \end{array}\right]- \\ 15.00\left[\begin{array}{c} X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+ \\ Y_{12}+Y_{22}+X_{32}+Y_{42} \end{array}\right] \end{array}\right] \ge 0.0$$

Upper limit:

For $n = 1$,
$$\left[\begin{array}{l}\left[\begin{array}{l}-3.071\times\begin{bmatrix}48.50X_{11}+46.50X_{21}+49.50X_{31}+50.05X_{41}+44.01X_{51}+\\ 11.27Y_{11}+0.72Y_{21}+2.99X_{31}+1.69Y_{41}\end{bmatrix}+\\ 8.600\times\begin{bmatrix}3.25X_{11}+5.01X_{21}+1.75X_{31}+2.50X_{41}+9.04X_{51}+\\ 51.51Y_{11}+76.69Y_{21}+58.28X_{31}+32.38Y_{41}\end{bmatrix}+\\ 5.068\times\begin{bmatrix}1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+\\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41}\end{bmatrix}+\\ 1.079\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\end{array}\right]-\\ 20.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\ Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\end{array}\right]\leq 0.0$$

For $n = 2$,
$$\left[\begin{array}{l}\left[\begin{array}{l}-3.071\times\begin{bmatrix}47.20X_{12}+45.30X_{22}+51.25X_{32}+52.08X_{42}+41.83X_{52}+\\ 10.18Y_{12}+0.79Y_{22}+2.63X_{32}+1.98Y_{42}\end{bmatrix}+\\ 8.600\times\begin{bmatrix}2.95X_{12}+5.21X_{22}+1.63X_{32}+2.69X_{42}+7.56X_{52}+\\ 52.00Y_{12}+75.30Y_{22}+59.13X_{32}+29.46Y_{42}\end{bmatrix}+\\ 5.068\times\begin{bmatrix}1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+\\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42}\end{bmatrix}+\\ 1.079\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\end{array}\right]-\\ 20.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\end{array}\right]\leq 0.0$$

6.2.12. C_3A constraint

Lower limit:

For $n = 1$,
$$\left[\begin{array}{l}\left[\begin{array}{l}2.650\times\begin{bmatrix}1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+\\ 12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41}\end{bmatrix}+\\ 1.692\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\ 5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\end{array}\right]-\\ 5.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\ Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\end{array}\right]\geq 0.0$$

For $n = 2$,
$$\left[\begin{array}{l}\left[\begin{array}{l}2.650\times\begin{bmatrix}1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+\\ 11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42}\end{bmatrix}+\\ 1.692\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\ 5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\end{array}\right]-\\ 5.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\ Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\end{array}\right]\geq 0.0$$

Upper limit:

For $n = 1$,
$$\left[\left[2.650\times\begin{bmatrix}1.83X_{11}+0.99X_{21}+1.50X_{31}+0.95X_{41}+1.99X_{51}+\\12.14Y_{11}+10.17Y_{21}+13.92X_{31}+13.07Y_{41}\end{bmatrix}+1.692\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\right]-8.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\le 0.0$$

For $n = 2$,
$$\left[\left[2.650\times\begin{bmatrix}1.96X_{12}+0.93X_{22}+1.42X_{32}+0.91X_{42}+1.86X_{52}+\\11.89Y_{12}+10.03Y_{22}+13.25X_{32}+12.69Y_{42}\end{bmatrix}+1.692\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\right]-8.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\le 0.0$$

6.2.13. C_4AF constraint

Lower limit:

For $n = 1$,
$$\left[\left[3.043\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\right]-5.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\ge 0.0$$

For $n = 2$,
$$\left[\left[3.043\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\right]-5.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\ge 0.0$$

Upper limit:

For $n = 1$,
$$\left[\left[3.043\times\begin{bmatrix}1.12X_{11}+1.25X_{21}+0.20X_{31}+1.05X_{41}+1.50X_{51}+\\5.35Y_{11}+5.74Y_{21}+6.53X_{31}+34.09Y_{41}\end{bmatrix}\right]-8.00\begin{bmatrix}X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+\\Y_{11}+Y_{21}+X_{31}+Y_{41}\end{bmatrix}\right]\le 0.0$$

For $n = 2$,
$$\left[\left[3.043\times\begin{bmatrix}1.15X_{12}+1.23X_{22}+0.19X_{32}+0.96X_{42}+1.65X_{52}+\\5.15Y_{12}+5.06Y_{22}+6.25X_{32}+36.24Y_{42}\end{bmatrix}\right]-8.00\begin{bmatrix}X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+\\Y_{12}+Y_{22}+X_{32}+Y_{42}\end{bmatrix}\right]\le 0.0$$

6.3. Solution of the LP formulation

The LP formulation for the development of two-period optimum production plan presented in the previous section consists of eighteen (18) decision variables representing the quantity (tons) of the raw materials to be mined from the limestone quarry and additives to be purchased from the market. The formulation also consists of one hundred (100) constraints, of these forty constraints are representing the quantity requirements and the remaining sixty (60) are representing quality requirements.

The problem requires analysis of one hundred and one (101) linear equations/functions of eighteen (18) variables. Therefore, manual solution to the problem is nearly impossible. As such, the solution is developed using Solver option (Add-in) in Microsoft Excel. It took only a couple of seconds for developing the optimum solution in Microsoft Excel.

The optimum solution minimized the combined cost of the two-period production plan to Rs. 718,659.00 by satisfying all quantity and quality requirements. The individual cost of supplying raw materials during periods 1 and 2 is Rs. 363,615.00 and Rs. 355,044.00, respectively. The production plan ensured supply of 15000 tons of raw mix to the cement plant during each period, therefore, the cost per ton during period 1 and 2 stands at Rs. 24.24 and Rs. 23.67, respectively. Table 6.7 shows the quantities of raw materials retrieved from the quarry and additives in each period. Similarly, the final values of the blending requirements are given in Table 6.8.

Table 6.7: Optimum raw materials production plan – quantity requirements

Material Location		Raw Mix (Tons)	
		Period 1	**Period 2**
Bench	1	1723	2000
	2	4500	3500
	3	3000	4000
	4	1000	1500
	5	1898	1000
Additive	Clay	2479	2321
	Slate	400	520
	Shale	0	159
	Laterite	0	0
Raw Mix (Tons)		15000	15000

Table 6.8: Optimum raw materials production plan – quality requirements

Chemicals/Quality Indicators	Percent Content	
	Period	
	1	2
CaO	40.21	40.16
SiO_2	14.10	14.10
Al_2O_3	3.40	3.40
Fe_2O_3	1.84	1.74
LOI	38.14	38.33
MgO	1.77	1.77
Na_2O	0.29	0.23
K_2O	0.26	0.27
Lime Saturation Factor (LSF)	0.90	0.90
Silica Ratio (SR)	2.69	2.75
Alumina Ratio (AM)	1.85	1.96
C_3S	31.09	31.00
C_2S	16.96	17.04
C_3A	5.90	6.07
C_4AF	5.60	5.28

As given in Table 6.7, for a raw mix stockpile accommodating a minimum of 15000 tons, the amount of raw materials contributed from the limestone quarry is 12,121 and 12,000 tons during periods 1 and 2, respectively. Therefore, the optimum production plan ensured maximum supply of raw materials from the quarry. It is also worth mentioning that the raw mix stockpile has been developed by avoiding the purchase of the most expensive additive i.e. lateraite. A layout of the LP formulation in Microsoft Excel and detailed description of the solution is given in Appendix A.

CHAPTER 7: SENSITIVITY ANALYSIS

7.1. Introduction

The sensitivity analysis of the LP model presented in the previous chapter allows convenient decision making to the management in the event of changing circumstances during the life of operation. For example, the management may have a compulsion to use at least 100 tons of laterite during each period as part of the contract signed with the supplier, or the quarry manager may have to produce only from benches 1 through 4 due to shortage of haulage equipment due to an alternative allocation.

The answer to these what if questions may either be developed through the solution report of the current LP model or by generating a new optimum solution of the modified LP model. As such, this chapter shares the analysis of different real life scenarios that may arise during implementation of the LP model.

7.2. Sensitivity report of the LP model

The Microsoft Excel Solver generates a sensitivity report as part of the optimum solution. The sensitivity report consists of two important economic parameters, the reduced cost of an unused activity (decision variable at a value equal to zero in the optimum solution) and the shadow price of a binding constraint.

The reduced cost per unit of an unused activity is the amount by which the objective function value may increase or decrease if the activity is forced into the solution. While shadow price per unit of a binding constraint depicts the amount by which the objective function value may increase or decrease, if the right hand side of a particular binding constraint is increased or decreased. For example, if the minimum quantity of available raw materials on bench 1 is consumed as a whole in the optimum solution, then the constraint representing this particular quantity requirement is a binding constraint. Because, the difference (slack) between the left hand side (tons consumed) and the right hand side (tons available) of this linear equation is equal to zero, leaving no flexibility (cushion) for the management. Therefore, an increase or decrease in the right hand side of this quantity requirement i.e. increase or decrease of minimum available tons of raw materials on bench 1 must have an impact (through a change in the optimum solution) on the objective function value.

Table 7.1 presents the reduced costs of the decision variables of the LP model presented in chapter 6.

Table 7.1: Reduced costs of the decision variables of the LP model

Period	Material Location		Symbol	Lower Limit (Tons)	Upper Limit (Tons)	Optimum (Tons)	Reduced Cost (Rs./ton)
1	Bench	1	X_{11}	1000	2500	1723	0
		2	X_{21}	3000	4500	4500	0
		3	X_{31}	3000	4000	3000	0
		4	X_{41}	1000	2500	1000	0
		5	X_{51}	1000	2500	1898	0
	Additive	Clay	Y_{11}	1500	3000	2479	0
		Slate	Y_{21}	400	1100	400	0
		Shale	Y_{31}	0	500	0	4
		Laterite	Y_{41}	0	150	0	352
2	Bench	1	X_{12}	2000	3000	2000	0
		2	X_{22}	2500	3500	3500	0
		3	X_{32}	4000	5000	4000	0
		4	X_{42}	1500	2500	1500	0
		5	X_{52}	1000	2500	1000	0
	Additive	Clay	Y_{12}	2000	4000	2321	0
		Slate	Y_{22}	400	1500	520	0
		Shale	Y_{32}	0	500	159	0
		Laterite	Y_{42}	0	150	0	360

As given in Table 7.1, the reduced costs of the unused (inactive) decision variables Y_{31}, Y_{41}, and Y_{42} are Rs. 4.00, Rs. 352.00, and Rs. 360 per ton of the raw materials, respectively. Therefore, if the management is bound to purchase at least 10 tons of laterite during period 1, then without solving the modified LP model that includes changed lower limit for this particular decision variable, the following changes in the optimum solution may be readily communicated:

Quantity of laterite in the raw mix = Y_{41} = 10 tons

Objective function (new value) = Rs. 722,179.00

Objective function (old value) = Rs. 718,659.00

Increase in objective function value = Rs. 3520.00 (Rs. 352/ton × 10 tons = Rs. 3520)
Same argument is also applicable to other inactive decision variables including Y_{31}, and Y_{42}. It is worth mentioning that the reduced cost of an active decision variable is always equal to zero.

Similarly, the shadow price of a non-binding constraint is always equal to zero. Table 7.2 displays the shadow prices of the binding quantity constraints of the LP formulation presented in chapter 6.

Table 7.2: Shadow prices of the binding quantity constraints of the LP model

Binding Constraint		Shadow Price (Rs./ton)
Quantity Constraints	Bench 2 – upper limit on tons of raw materials during period 1 “X_{21}”	-6.00
	Bench 3 – lower limit on tons of raw materials during period 1 “X_{31}”	5.00
	Bench 4 – lower limit on tons of raw materials during period 1 “X_{41}”	2.00
	Bench 1 – lower limit on tons of raw materials during period 2 “X_{12}”	4.00
	Bench 2 – upper limit on tons of raw materials during period 2 “X_{22}”	-7.00
	Bench 3 – lower limit on tons of raw materials during period 2 “X_{31}”	11.00
	Bench 4 – lower limit on tons of raw materials during period 2 “X_{42}”	5.00
	Bench 5 – lower limit on tons of raw materials during period 2 “X_{52}”	3.00
	Slate - upper limit on tons of raw materials during period 1 “Y_{21}”	9.00
	Raw mix stockpile capacity constraint during period 1	25.00
	Raw mix stockpile capacity constraint during period 2	21.00

A shadow price of Rs. -6.00 per ton for the upper limit on tons of raw materials from bench 2 during period 1 shows that an increase of 1 unit in the right hand side of this constraint will reduce the objective function value by Rs. 6.00. Similarly, an increase of 1 unit in the right hand side of the constraint representing lower limit on the tons of raw materials from bench 3 during period 1 shall increase the objective function by Rs. 5.00.

In essence, the sensitivity report containing the reduced costs of decision variables and the shadow prices of the constraints proves to be a help in answering the what if questions, especially, in case of large scale optimization problems.

7.3. Modified LP formulations

The proposed modifications in LP formulation are only in quantity constraints. Of course, the objective function and quality constraints may not be compromised. The purpose of the sensitivity analysis is to explore the impact of modifications in quantity constraints such that the sustained supply of raw materials is ensured under all real life scenarios/circumstances.

Two alternatives are explored here; one is the periodic mandatory maintenance of the production equipment being utilized on any of the five quarry benches and second is the periodic development work on benches 1 and 2 for allowing exposure of more potential production areas on benches 3, 4 and 5.

The maintenance alternative takes the equipment solely out of production from a particular bench in a given period leading the raw materials production to zero. However, the development alternative takes the equipment from bench 3, 4, or 5 and utilizes the same at benches 1 and 2. As such, the production from bench 3, 4, or 5 goes to zero by relative enhancement in the raw materials production from benches 1 and 2. Table 7.3 and Figs. 7.1 through 7.6 present the impact of maintenance alternative on the objective function.

Table 7.3: Sensitivity analysis considering the equipment maintenance alternative*

Bench	Maximum Raw Materials Production (Tons)		Total Cost	Cost per Ton	
	Period 1	Period 2	(Rs.)	Period 1	Period 2
1	0	2000	727027.23	24.80	23.67
	1000	0	764139.44	24.77	24.15
	0	0	734071.52	24.80	24.14
2	2000	3500	738662.97	25.57	23.67
	4000	1200	747493.64	24.44	25.39
	2000	1200	764448.10	25.57	25.39
3	250	4000	719090.43	24.27	23.67
	3000	1500	720807.53	24.24	23.81
	250	1500	721238.55	24.27	23.81
4	0	1500	716650.39	24.11	23.67
	1000	0	762883.97	24.57	24.30
	0	0	726091.78	24.11	24.30
5	0	1000	722548.86	24.50	23.67
	1898	0	718638.36	24.24	23.67
	0	0	722527.77	24.50	23.67

* Detailed description of the solutions in MS Excel is given in Appendix A from pages 69 to 84

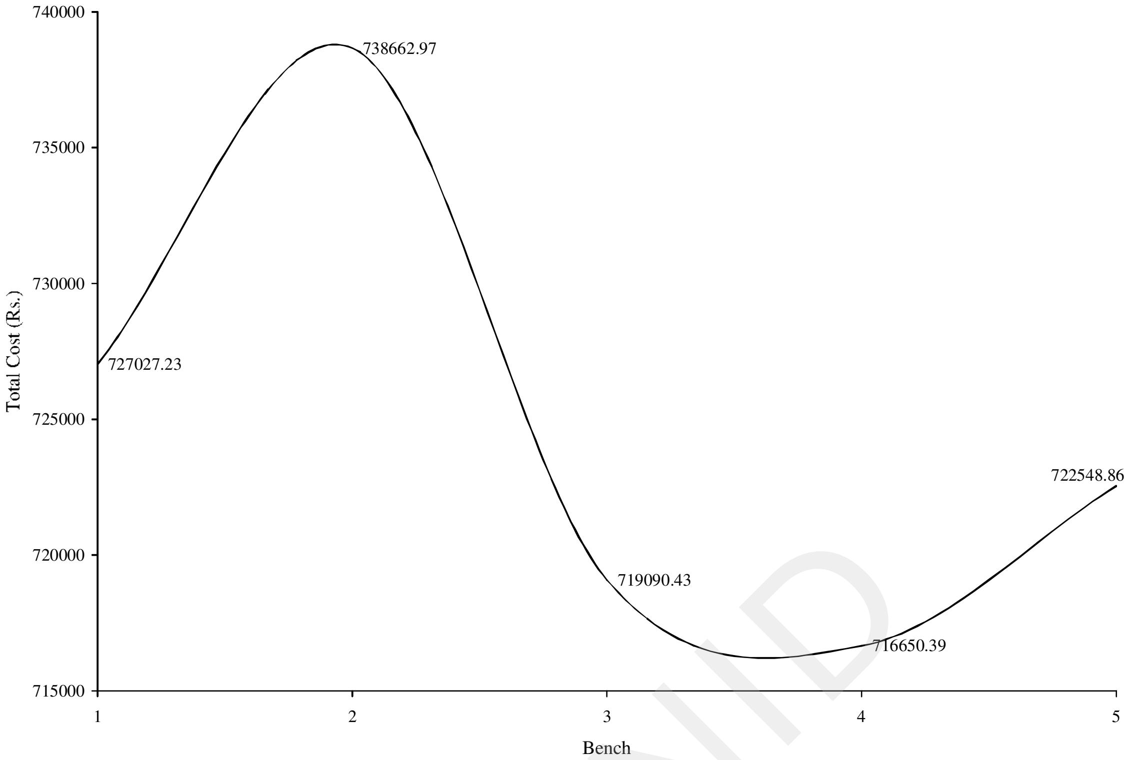


Fig. 7.1: Sensitivity of the total cost during equipment maintenance during period 1

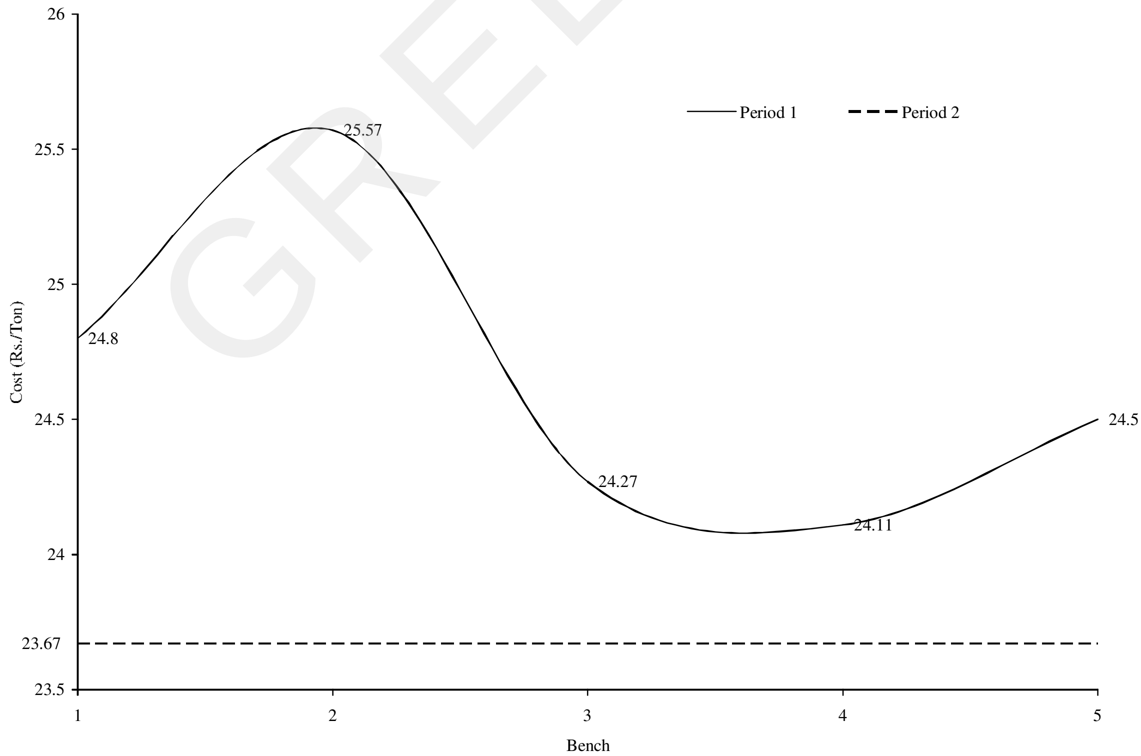


Fig. 7.2: Sensitivity of the cost per ton during equipment maintenance during period 1

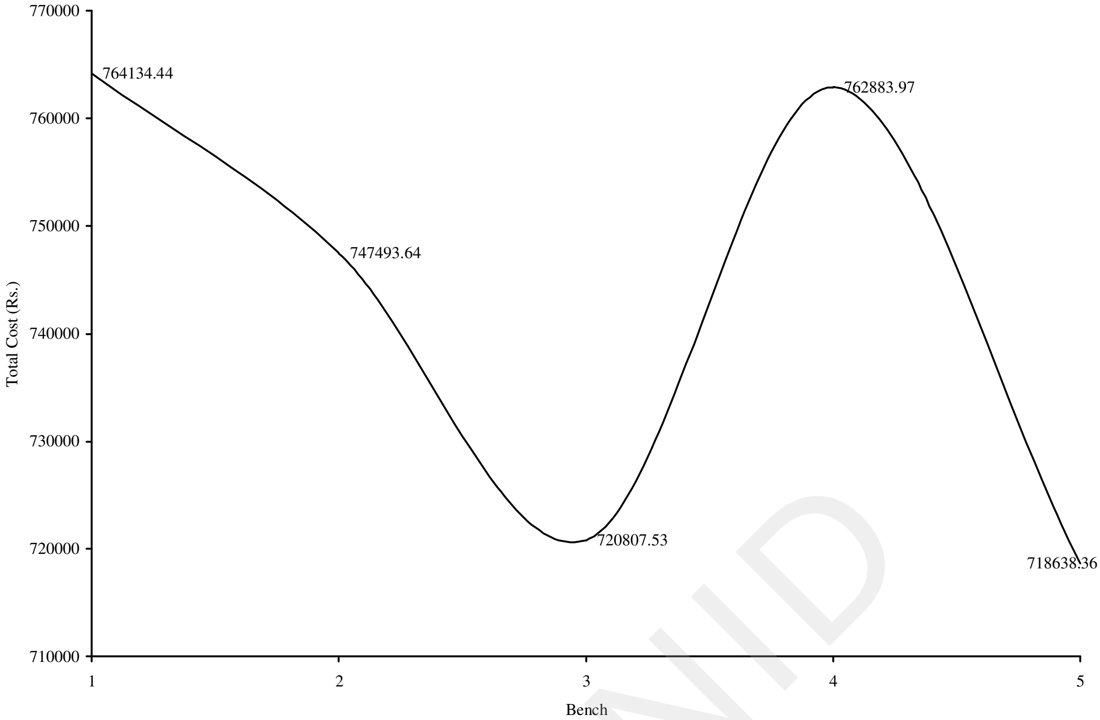


Fig. 7.3: Sensitivity of the total cost during equipment maintenance during period 2

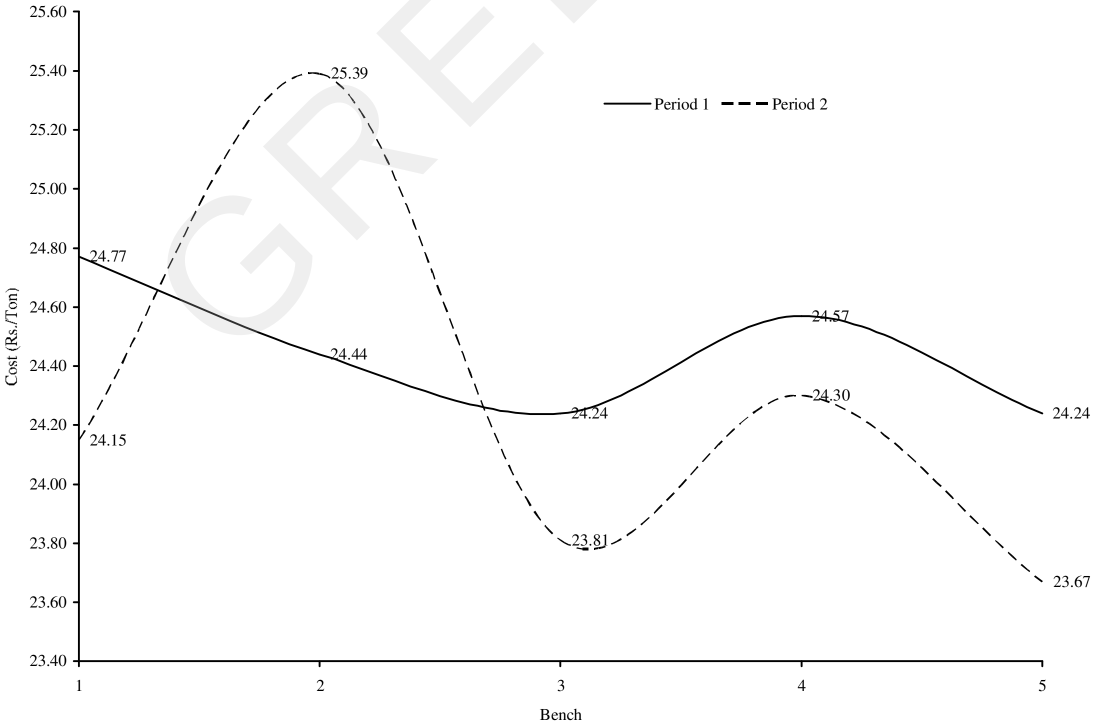


Fig. 7.4: Sensitivity of the cost per ton during equipment maintenance during period 2

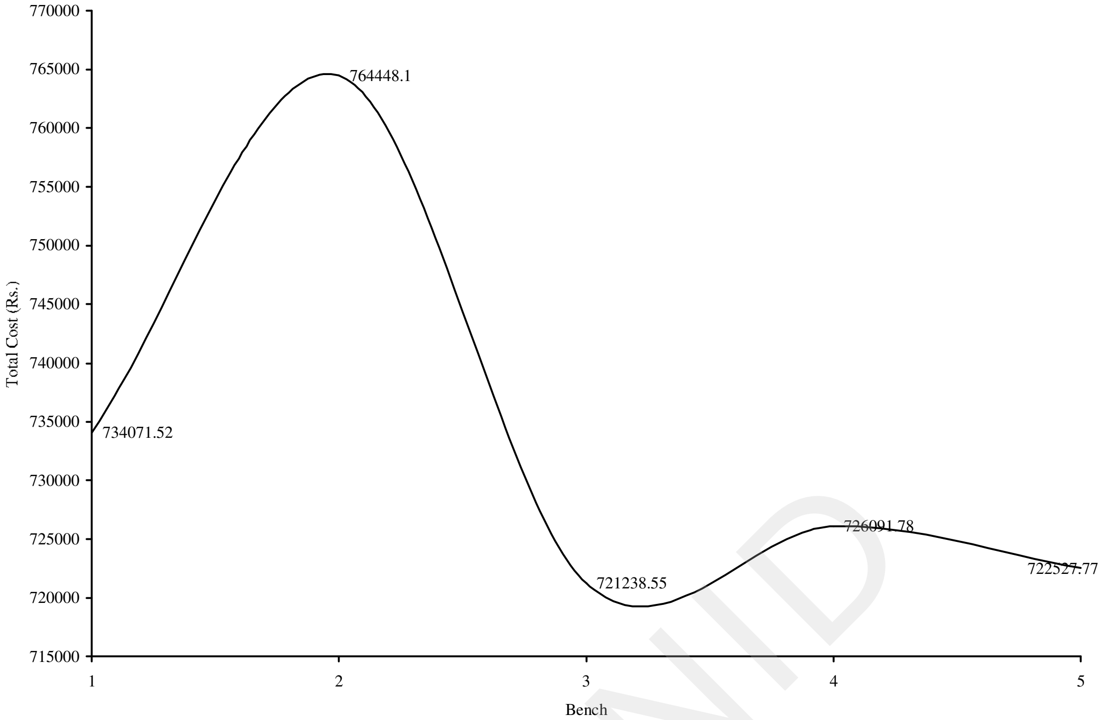


Fig. 7.5: Sensitivity of the total cost during equipment maintenance during period 1 and 2

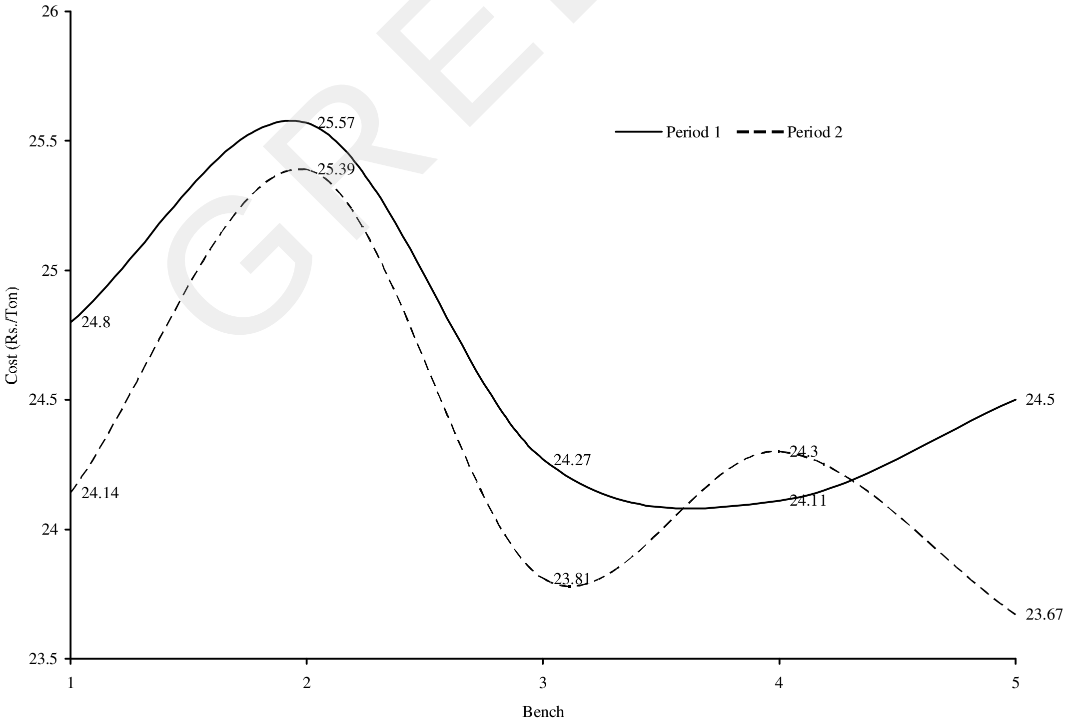


Fig. 7.6: Sensitivity of the cost per ton during equipment maintenance during periods 1 and 2

As given in Table 7.3 and Figs. 7.1 through 7.6, the most economical alternative for equipment maintenance at bench 1 is during both periods 1 and 2. Similarly, in order to satisfy the quality constraints, benches 2 and 3 must always contribute raw materials for the development of optimum raw mix; as such their production may never be equal to zero. Therefore, the equipment from other benches shall be coordinated for a minimum production from benches 2 and 3 while their dedicated equipment is being maintained. The production plan is least expensive when equipment maintenance at benches 2 and 3 is conducted during periods 1 and 2, respectively. Also, for benches 4 and 5 the least expensive production plans are during periods 1 and 2, respectively.

It is worth mentioning that the number of benches dictate the completion of quarry equipment maintenance requires more than one two-period short-range production plans. However, a five-period (equal to number of benches) short-range production plan will accommodate a complete quarry equipment maintenance schedule.

Table 7.4 and Figs 7.7 through 7.12 present the impact of development alternative on the objective function of the optimum two-period short-range production plan.

Table 7.4: Sensitivity analysis considering the development alternative†

Contributing Bench	**Raw Materials Production at Contributing Bench (Tons)**		**Raw Materials Production (Tons)**				**Total Cost (Rs.)**	**Cost per Ton**	
	Period		**Period 1**		**Period 2**			**Period**	
	1	**2**	**Bench 1**	**Bench 2**	**Bench 1**	**Bench 2**		**1**	**2**
3	0	4000	4500	5623	2000	3500	700191.91	23.01	23.67
	3731	0	1712	3719	4731	5213	833716.22	28.05	23.56
	0	0	3763	4690	5500	4500	741405.93	23.87	23.27
4	0	1500	2293	5750	2000	3500	709012.56	23.60	23.67
	1000	0	1723	4500	2750	3734	727432.64	24.24	24.25
	0	0	2293	5750	2750	3734	717785.81	23.60	23.25
5	0	1000	2226	5750	2000	3500	711680.41	23.77	23.67
	1898	0	1723	4500	2500	3986	713377.87	24.24	23.32
	0	0	2226	5750	2500	3986	706398.79	23.78	23.32

† Detailed description of the solutions in MS Excel is given in Appendix A from pages 85 to 93

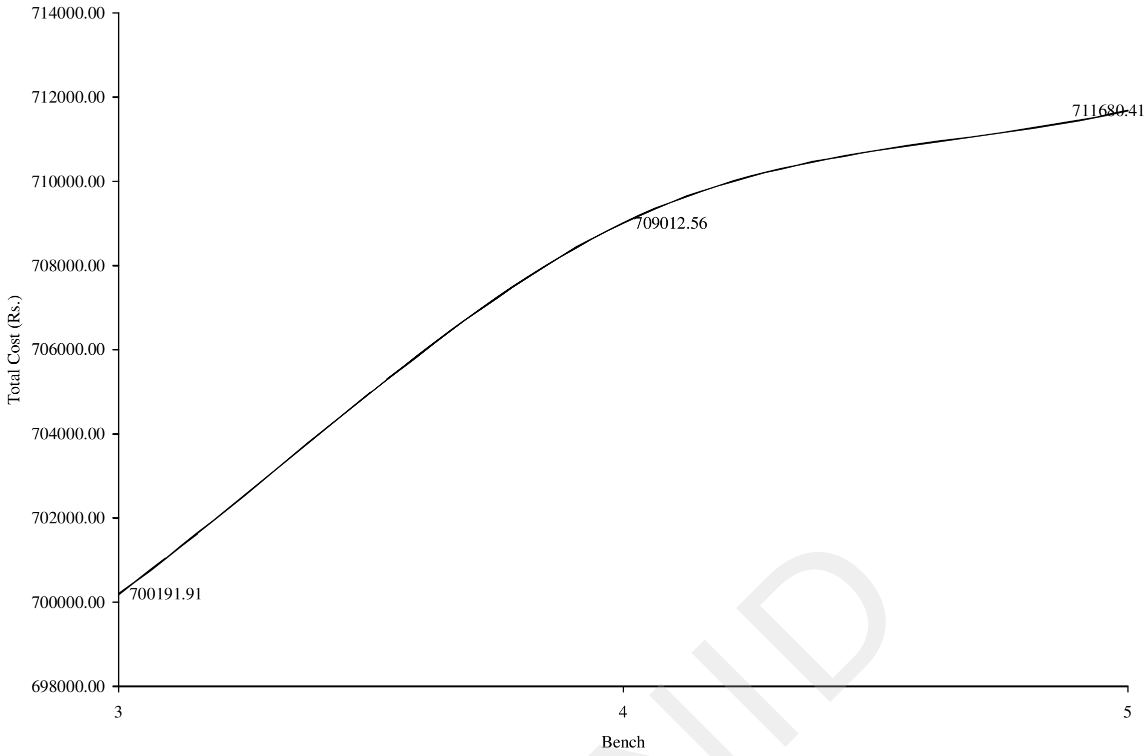


Fig. 7.7: Sensitivity of the total cost for development alternative with zero production in period 1

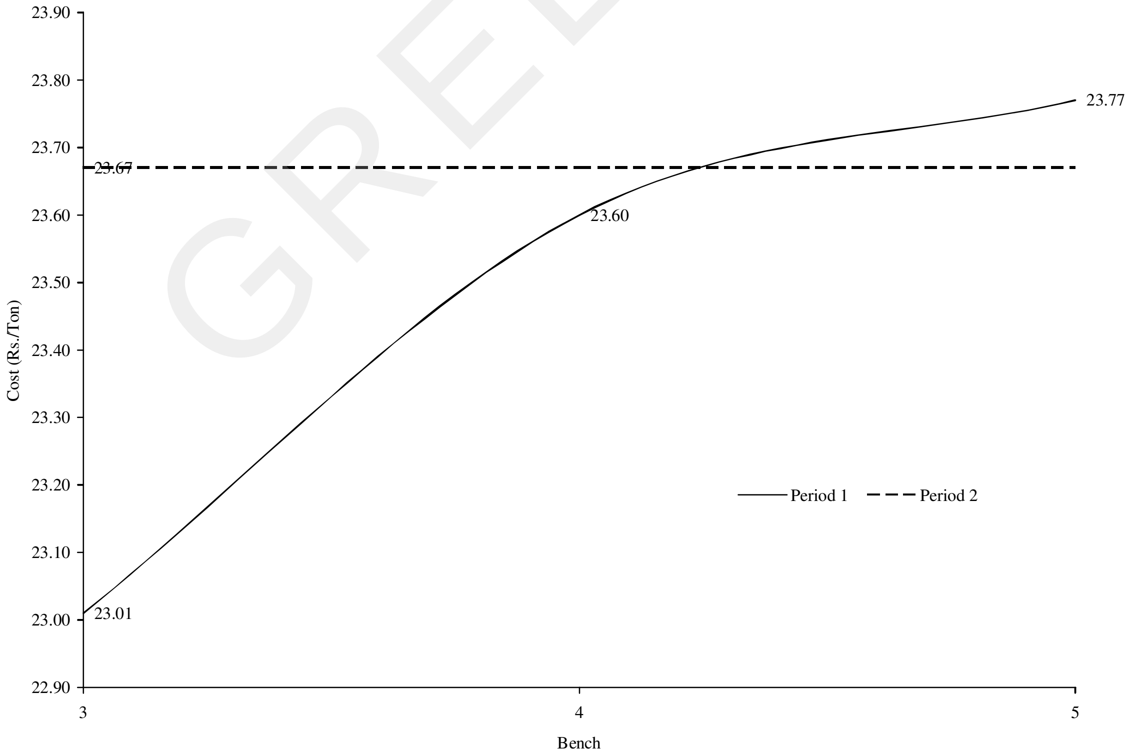


Fig. 7.8: Sensitivity of the cost per ton for development alternative with zero production in period 1

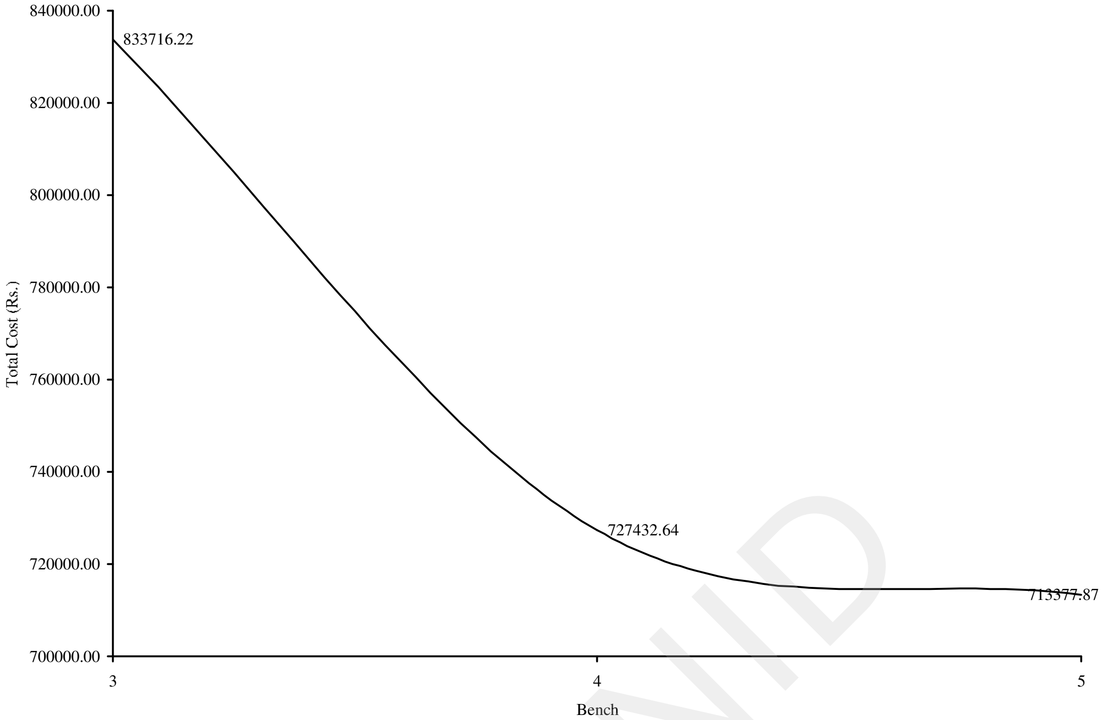


Fig. 7.9: Sensitivity of the total cost for development alternative with zero production in period 2

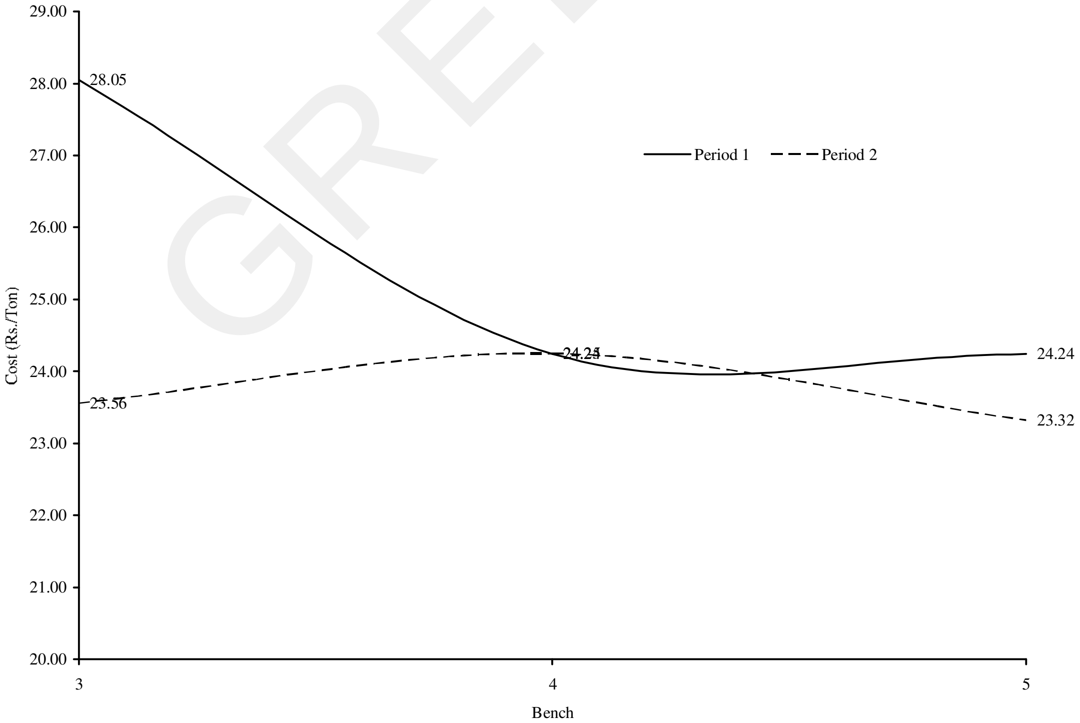


Fig. 7.10: Sensitivity of the cost per ton for development alternative with zero production in period 2

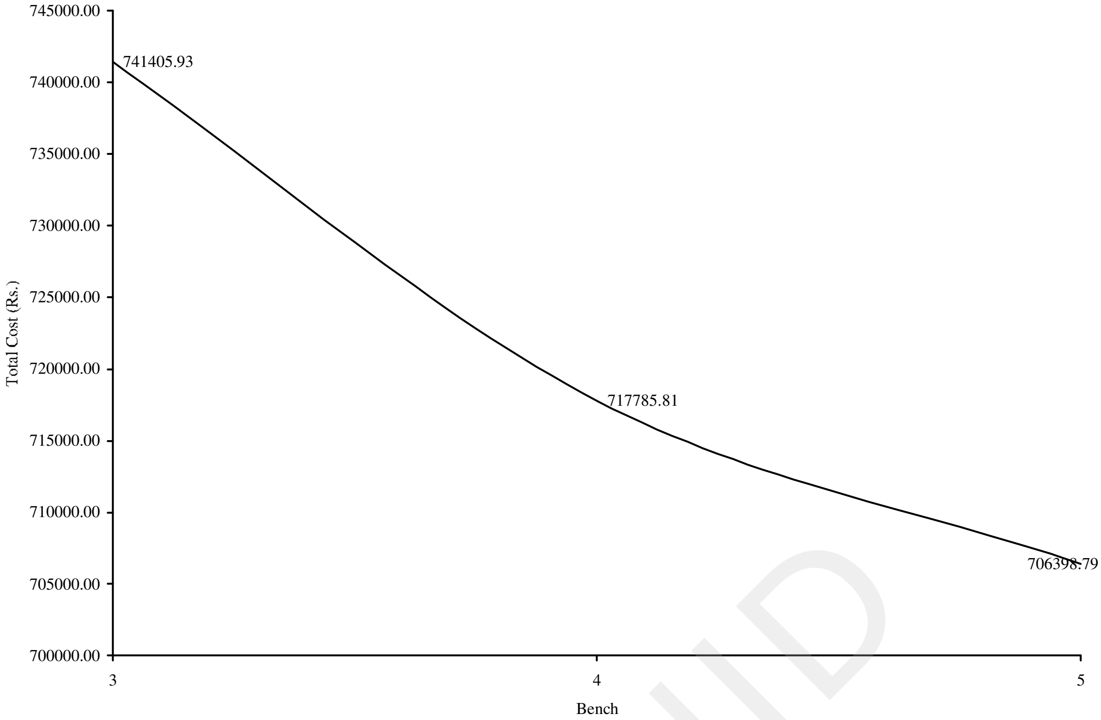


Fig. 7.11: Sensitivity of the total cost for development alternative with zero production in both periods

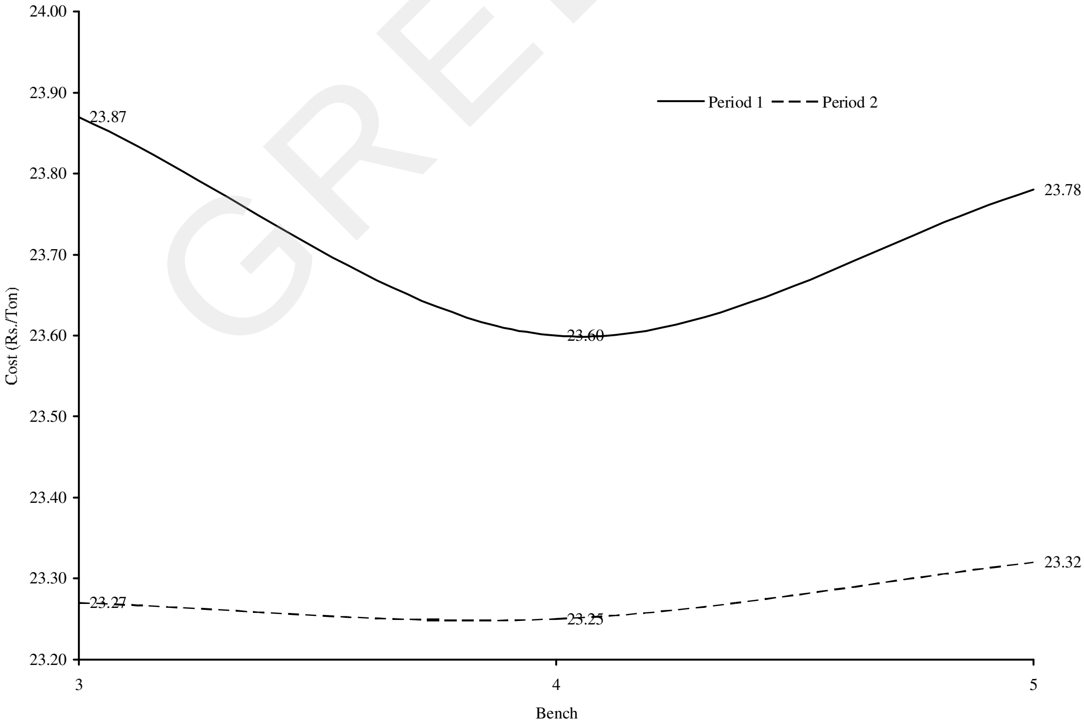


Fig. 7.12: Sensitivity of the cost per ton for development alternative with zero production in both periods

As given in Table 7.4 and Figs. 7.7 through 7.12, the least cost alternative for executing the mandatory development on upper benches of the quarry comes from bench 3 contributing its production capacity during period 1. Similarly, depending upon the intensity of the development work, bench 5 also offers its production capacity in both periods at the minimum possible total as well as per ton costs.

As mentioned earlier, the sensitivity analysis by answering what if questions, promises to enhance the management's perception with respect to the performance of various stages of the cement manufacturing operation. This chapter presented few alternatives; however, several other production planning issues may be incorporated into the sensitivity analysis for resolving real life scenarios.

CHAPTER 8: CONCLUSIONS & RECOMMENDATIONS

8.1. Conclusions

Optimum short-range production planning of the cement quarry operations has been a challenge for the mining industry. In contrast to the manual trial and error approach, this research has attempted to contribute an optimum solution to this large scale and complex optimization problem.

Cement manufacturing operations heavily depend upon the accurate blending of raw materials for producing an acceptable quality final product. Therefore, the industry needs a proven scientific approach to develop a raw mix stockpile consisting of the raw materials from the limestone quarry and the additives. The methodology presented in this study has demonstrated to overcome the disadvantages associated with the trial and error approach. The trial and error approach normally leads to lack of coordination between the limestone quarry and the quality control departments of a cement manufacturing operation, and in other words, it causes mismanagement and inefficient use of valuable resources.

Cost minimization is the objective function of the linear programming based mathematical model. As such, the solution to the basic LP formulation for an existing cement manufacturing operation in northern Pakistan led to the satisfaction of all quantity and quality constraints for a combined cost of Rs. 718,659.00 in a two-period production plan. The optimum production plan ensured supply of 15000 tons of raw mix to the cement plant during each period, therefore, the cost per ton during period 1 and 2 stands at Rs. 24.24 and Rs. 23.67, respectively.

As compared to the existing production plan (developed using trial and error approach), the optimum production plan promised an approximate cost saving of Rs. 8 million per year to this cement manufacturing operation. One important aspect of the mathematical programming based optimization models is their implementation. However, enormous cost savings along with continuous involvement and interest of the plant management throughout this research ensured its implementation. The execution of LP model in Microsoft Excel as opposed to expensive optimization solvers also guaranteed its effortless implementation.

Sensitivity analysis provides additional flexibility for evaluating multiple planning alternatives (horizons) under existing circumstances through simple modifications of the LP formulations. Therefore, keeping in view the established research objectives, the short-range production scheduling model presented in this study offers flexibility for sound decision making with respect to production planning and adequate engineering and operational control.

8.2. Recommendations for future research

The linear programming model may be modified into a mixed integer linear programming (MILP) formulation accommodating blocks on various benches/zones of the limestone quarry as integer (0/1) variables. Therefore, if a block is mined in a given period, it is assigned a value equal to one (1); otherwise its value is equal to zero (0).

The introduction of the integer variables into the formulation increases solution time exponentially. Nevertheless, short-range production planning requires less integer variables; therefore, with mere increase in computational cost, MILP formulation offers a positive improvement towards the implementation of the mathematical models because of their closeness to reality.

Implementation of this research also led to another successful liaison with one of the most modern cement manufacturing operations in Pakistan. The data from this operation is being utilized in an ongoing PhD research under my supervision, which is considering the incorporation of integer variables for the development of optimum short-range production plans.

Complete automation of this large scale optimization problem through implementation of the algorithmic steps in a .Net programming language will be another promising contribution to the cement industry. Especially, this will prove its worth for the development of multi-period (more than two) production plans.

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APPENDIX A: MICROSOFT EXCEL SOLUTIONS

MS Excel solution for LP problem presented in chapter 6

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
1723	4500	3000	1000	1898	2479	400	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12121									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Mix Vlaue	Minimum	Maximum	Mix Value
40.000	42.000	40.21	40.000	42.000	40.16
14.000	15.000	14.10	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.74
35.000	40.000	38.14	35.000	40.000	38.33
0.000	2.000	1.77	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.75
1.500	2.000	1.85	1.500	2.000	1.96
30.000	35.000	31.09	30.000	35.000	31.00
15.000	20.000	16.96	15.000	20.000	17.04
5.000	8.000	5.90	5.000	8.000	6.07
5.000	8.000	5.60	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	718659.45	
	Period 1	Period 2
Cost (Rs./ton)	24.24066778	23.66996242

ion for modified LP problem in equipment maintenance alternative with zero production at bench 1 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
0	3000	3000	1000	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
0	4500	4000	2500	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
0	4500	4000	2015	1481	2604	400	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
11996									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.19	40.000	42.000	40.16
14.000	15.000	14.19	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.80	1.650	2.170	1.74
35.000	40.000	37.87	35.000	40.000	38.33
0.000	2.000	2.00	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.73	2.600	2.900	2.75
1.500	2.000	1.89	1.500	2.000	1.96
30.000	35.000	30.40	30.000	35.000	31.00
15.000	20.000	17.74	15.000	20.000	17.04
5.000	8.000	5.97	5.000	8.000	6.07
5.000	8.000	5.48	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	727027.23	
	Period 1	Period 2
Cost (Rs./ton)	24.79851919	23.66996257

ation for modified LP problem in equipment maintenance alternative with zero production at bench 1 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	0	2500	4000	1500	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	0	3500	5000	2500	2500	4000	1500	500	150
1000	4498	3998	1997	1476	2651	500	40	0	0	3498	4852	2498	1058	2760	400	0	0
15000									15000								
12969									11906								
15000									15000								
16160									15066								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.27	40.000	42.000	40.48
14.000	15.000	14.13	14.000	15.000	14.24
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.80	1.650	2.170	1.70
35.000	40.000	37.91	35.000	40.000	37.69
0.000	2.000	1.93	0.000	2.000	1.97
0.000	0.500	0.26	0.000	0.500	0.25
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.71	2.600	2.900	2.79
1.500	2.000	1.88	1.500	2.000	2.00
30.000	35.000	31.15	30.000	35.000	31.33
15.000	20.000	17.01	15.000	20.000	17.18
5.000	8.000	5.96	5.000	8.000	6.13
5.000	8.000	5.49	5.000	8.000	5.17

Objective Function		
Total Cost (Rs.)	764139.44	
	Period 1	Period 2
Cost (Rs./ton)	24.77319386	24.14730772

n for modified LP problem in equipment maintenance alternative with zero production at bench 1 during periods 1 & 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
0	3000	3000	1000	1000	1500	400	0	0	0	2500	4000	1500	1000	2000	400	0	0
0	4500	4000	2500	2500	3000	1100	500	150	0	3500	5000	2500	2500	4000	1500	500	150
0	4500	4000	2015	1481	2604	400	0	0	0	3500	4825	2500	1024	2750	400	0	0
15000									15000								
11996									11850								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.19	40.000	42.000	40.48
14.000	15.000	14.19	14.000	15.000	14.25
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.80	1.650	2.170	1.70
35.000	40.000	37.87	35.000	40.000	37.67
0.000	2.000	2.00	0.000	2.000	1.98
0.000	0.500	0.26	0.000	0.500	0.25
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.73	2.600	2.900	2.79
1.500	2.000	1.89	1.500	2.000	2.00
30.000	35.000	30.40	30.000	35.000	31.25
15.000	20.000	17.74	15.000	20.000	17.27
5.000	8.000	5.97	5.000	8.000	6.13
5.000	8.000	5.48	5.000	8.000	5.17

Objective Function		
Total Cost (Rs.)	734071.52	
	Period 1	Period 2
Cost (Rs./ton)	24.79851919	24.13958191

n for modified LP problem in equipment maintenance alternative with minimum production at bench 2 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	2000	3000	1000	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
2500	2000	4000	2500	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2500	2000	3000	2184	2500	2161	655	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12184									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.46	40.000	42.000	40.16
14.000	15.000	14.20	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.82	1.650	2.170	1.74
35.000	40.000	37.83	35.000	40.000	38.33
0.000	2.000	1.74	0.000	2.000	1.77
0.000	0.500	0.27	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.72	2.600	2.900	2.75
1.500	2.000	1.87	1.500	2.000	1.96
30.000	35.000	31.35	30.000	35.000	31.00
15.000	20.000	17.07	15.000	20.000	17.04
5.000	8.000	5.93	5.000	8.000	6.07
5.000	8.000	5.53	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	738662.97	
	Period 1	Period 2
Cost (Rs./ton)	25.57423548	23.66996274

for modified LP problem in equipment maintenance alternative with minimum production at bench 2 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2000	1200	4000	1500	1000	2000	400	0	0
2500	4000	4000	2500	2500	3000	1100	500	150	3000	1200	5000	2500	2500	4000	1500	500	150
1688	4000	3000	1000	2483	2429	400	0	0	2947	1200	4000	1500	2500	2042	811	0	0
15000									15000								
12171									12147								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.22	40.000	42.000	40.17
14.000	15.000	14.10	14.000	15.000	14.11
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.72
35.000	40.000	38.17	35.000	40.000	38.54
0.000	2.000	1.72	0.000	2.000	1.56
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.76
1.500	2.000	1.85	1.500	2.000	1.98
30.000	35.000	31.10	30.000	35.000	31.01
15.000	20.000	16.97	15.000	20.000	17.06
5.000	8.000	5.90	5.000	8.000	6.10
5.000	8.000	5.59	5.000	8.000	5.24

Objective Function		
Total Cost (Rs.)	747493.64	
	Period 1	Period 2
Cost (Rs./ton)	24.44393809	25.38897158

...r modified LP problem in equipment maintenance alternative with minimum production at bench 2 during periods 1 & 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	2000	3000	1000	1000	1500	400	0	0	2000	1200	4000	1500	1000	2000	400	0	0
2500	2000	4000	2500	2500	3000	1100	500	150	3000	1200	5000	2500	2500	4000	1500	500	150
2500	2000	3000	2184	2500	2161	655	0	0	2947	1200	4000	1500	2500	2042	811	0	0
15000									15000								
12184									12147								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.46	40.000	42.000	40.17
14.000	15.000	14.20	14.000	15.000	14.11
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.82	1.650	2.170	1.72
35.000	40.000	37.83	35.000	40.000	38.54
0.000	2.000	1.74	0.000	2.000	1.56
0.000	0.500	0.27	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.72	2.600	2.900	2.76
1.500	2.000	1.87	1.500	2.000	1.98
30.000	35.000	31.35	30.000	35.000	31.01
15.000	20.000	17.07	15.000	20.000	17.06
5.000	8.000	5.93	5.000	8.000	6.10
5.000	8.000	5.53	5.000	8.000	5.24

Objective Function		
Total Cost (Rs.)	764448.10	
	Period 1	Period 2
Cost (Rs./ton)	25.57423548	25.38897125

n for modified LP problem in equipment maintenance alternative with minimum production at bench 3 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	250	1000	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
2500	4500	250	2500	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2500	4500	250	2500	2445	2405	400	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12195									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.20	40.000	42.000	40.16
14.000	15.000	14.27	14.000	15.000	14.10
2.700	3.400	3.33	2.700	3.400	3.40
1.650	2.170	2.00	1.650	2.170	1.74
35.000	40.000	37.91	35.000	40.000	38.33
0.000	2.000	1.75	0.000	2.000	1.77
0.000	0.500	0.27	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.89	0.845	0.900	0.90
2.600	2.900	2.68	2.600	2.900	2.75
1.500	2.000	1.67	1.500	2.000	1.96
30.000	35.000	30.00	30.000	35.000	31.00
15.000	20.000	18.28	15.000	20.000	17.04
5.000	8.000	5.44	5.000	8.000	6.07
5.000	8.000	6.07	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	719090.43	
	Period 1	Period 2
Cost (Rs./ton)	24.26940006	23.66996215

for modified LP problem in equipment maintenance alternative with minimum production at bench 3 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2000	2500	1500	1500	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	3000	3500	1500	2500	2500	4000	1500	500	150
1723	4500	3000	1000	1898	2479	400	0	0	3000	3500	1500	2500	1681	2153	666	0	0
15000									15000								
12121									12181								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.21	40.000	42.000	40.00
14.000	15.000	14.10	14.000	15.000	14.07
2.700	3.400	3.40	2.700	3.400	3.26
1.650	2.170	1.84	1.650	2.170	1.84
35.000	40.000	38.14	35.000	40.000	38.63
0.000	2.000	1.77	0.000	2.000	1.67
0.000	0.500	0.26	0.000	0.500	0.25
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.75
1.500	2.000	1.85	1.500	2.000	1.77
30.000	35.000	31.09	30.000	35.000	31.35
15.000	20.000	16.96	15.000	20.000	16.69
5.000	8.000	5.90	5.000	8.000	5.53
5.000	8.000	5.60	5.000	8.000	5.61

Objective Function		
Total Cost (Rs.)	720807.53	
	Period 1	Period 2
Cost (Rs./ton)	24.24066457	23.81317063

for modified LP problem in equipment maintenance alternative with minimum production at bench 3 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	250	1000	1000	1500	400	0	0	2000	2500	1500	1500	1000	2000	400	0	0
2500	4500	250	2500	2500	3000	1100	500	150	3000	3500	1500	2500	2500	4000	1500	500	150
2500	4500	250	2500	2445	2405	400	0	0	3000	3500	1500	2500	1681	2153	666	0	0
15000									15000								
12195									12181								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.20	40.000	42.000	40.00
14.000	15.000	14.27	14.000	15.000	14.07
2.700	3.400	3.33	2.700	3.400	3.26
1.650	2.170	2.00	1.650	2.170	1.84
35.000	40.000	37.91	35.000	40.000	38.63
0.000	2.000	1.75	0.000	2.000	1.67
0.000	0.500	0.27	0.000	0.500	0.25
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.89	0.845	0.900	0.90
2.600	2.900	2.68	2.600	2.900	2.75
1.500	2.000	1.67	1.500	2.000	1.77
30.000	35.000	30.00	30.000	35.000	31.35
15.000	20.000	18.28	15.000	20.000	16.69
5.000	8.000	5.44	5.000	8.000	5.53
5.000	8.000	6.07	5.000	8.000	5.61

Objective Function		
Total Cost (Rs.)	721238.55	
Cost (Rs./ton)	Period 1	Period 2
	24.26940006	23.81316994

ion for modified LP problem in equipment maintenance alternative with zero production at bench 4 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	0	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
2500	4500	4000	0	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2215	4500	3000	0	2500	2383	402	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12215									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.16	40.000	42.000	40.16
14.000	15.000	14.08	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.83	1.650	2.170	1.74
35.000	40.000	38.38	35.000	40.000	38.33
0.000	2.000	1.61	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.75
1.500	2.000	1.85	1.500	2.000	1.96
30.000	35.000	31.03	30.000	35.000	31.00
15.000	20.000	16.95	15.000	20.000	17.04
5.000	8.000	5.91	5.000	8.000	6.07
5.000	8.000	5.58	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	716650.41	
	Period 1	Period 2
Cost (Rs./ton)	24.10672924	23.66996506

ion for modified LP problem in equipment maintenance alternative with zero production at bench 4 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2000	2500	4000	0	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	3000	3500	5000	0	2500	4000	1500	500	150
2194	4500	3000	1000	2500	2449	513	58	0	3000	3500	4604	0	1000	2108	785	0	4
15000									15000								
13194									12104								
15000									15000								
16215									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.23	40.000	42.000	40.00
14.000	15.000	14.12	14.000	15.000	14.06
2.700	3.400	3.37	2.700	3.400	3.37
1.650	2.170	1.84	1.650	2.170	1.68
35.000	40.000	38.19	35.000	40.000	38.79
0.000	2.000	1.69	0.000	2.000	1.62
0.000	0.500	0.26	0.000	0.500	0.21
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.71	2.600	2.900	2.78
1.500	2.000	1.83	1.500	2.000	2.00
30.000	35.000	31.22	30.000	35.000	30.93
15.000	20.000	16.92	15.000	20.000	16.99
5.000	8.000	5.81	5.000	8.000	6.08
5.000	8.000	5.61	5.000	8.000	5.12

Objective Function		
Total Cost (Rs.)	762883.97	
	Period 1	Period 2
Cost (Rs./ton)	24.56980816	24.29938972

for modified LP problem in equipment maintenance alternative with zero production at bench 4 during periods 1 & 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	0	1000	1500	400	0	0	2000	2500	4000	0	1000	2000	400	0	0
2500	4500	4000	0	2500	3000	1100	500	150	3000	3500	5000	0	2500	4000	1500	500	150
2215	4500	3000	0	2500	2383	402	0	0	3000	3500	4604	0	1000	2108	785	0	4
15000									15000								
12215									12104								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.16	40.000	42.000	40.00
14.000	15.000	14.08	14.000	15.000	14.06
2.700	3.400	3.40	2.700	3.400	3.37
1.650	2.170	1.83	1.650	2.170	1.68
35.000	40.000	38.38	35.000	40.000	38.79
0.000	2.000	1.61	0.000	2.000	1.62
0.000	0.500	0.26	0.000	0.500	0.21
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.78
1.500	2.000	1.85	1.500	2.000	2.00
30.000	35.000	31.03	30.000	35.000	30.93
15.000	20.000	16.95	15.000	20.000	16.99
5.000	8.000	5.91	5.000	8.000	6.08
5.000	8.000	5.58	5.000	8.000	5.12

Objective Function		
Total Cost (Rs.)	726091.78	
	Period 1	Period 2
Cost (Rs./ton)	24.10672974	24.29938919

ion for modified LP problem in equipment maintenance alternative with zero production at bench 5 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	0	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
2500	4500	4000	2500	0	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2500	4500	3000	1969	0	2423	609	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
11969									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.35	40.000	42.000	40.16
14.000	15.000	14.15	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.74
35.000	40.000	37.76	35.000	40.000	38.33
0.000	2.000	1.95	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.70	2.600	2.900	2.75
1.500	2.000	1.85	1.500	2.000	1.96
30.000	35.000	31.24	30.000	35.000	31.00
15.000	20.000	17.01	15.000	20.000	17.04
5.000	8.000	5.90	5.000	8.000	6.07
5.000	8.000	5.59	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	722548.86	
	Period 1	Period 2
Cost (Rs./ton)	24.49996139	23.66996273

ion for modified LP problem in equipment maintenance alternative with zero production at bench 5 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2000	2500	4000	1500	0	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	3000	3500	5000	2500	0	4000	1500	500	150
1723	4500	3000	1000	1898	2479	400	0	0	2951	3500	4000	1500	0	2331	718	0	0
15000									15000								
12121									11951								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.21	40.000	42.000	40.35
14.000	15.000	14.10	14.000	15.000	14.18
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.70
35.000	40.000	38.14	35.000	40.000	38.08
0.000	2.000	1.77	0.000	2.000	1.80
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.78
1.500	2.000	1.85	1.500	2.000	2.00
30.000	35.000	31.09	30.000	35.000	31.19
15.000	20.000	16.96	15.000	20.000	17.13
5.000	8.000	5.90	5.000	8.000	6.13
5.000	8.000	5.60	5.000	8.000	5.18

Objective Function		
Total Cost (Rs.)	718638.36	
	Period 1	Period 2
Cost (Rs./ton)	24.24066617	23.66855792

n for modified LP problem in equipment maintenance alternative with zero production at bench 5 during periods 1 & 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	0	1500	400	0	0	2000	2500	4000	1500	0	2000	400	0	0
2500	4500	4000	2500	0	3000	1100	500	150	3000	3500	5000	2500	0	4000	1500	500	150
2500	4500	3000	1969	0	2423	609	0	0	2951	3500	4000	1500	0	2331	718	0	0
15000									15000								
11969									11951								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.35	40.000	42.000	40.35
14.000	15.000	14.15	14.000	15.000	14.18
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.70
35.000	40.000	37.76	35.000	40.000	38.08
0.000	2.000	1.95	0.000	2.000	1.80
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.70	2.600	2.900	2.78
1.500	2.000	1.85	1.500	2.000	2.00
30.000	35.000	31.24	30.000	35.000	31.19
15.000	20.000	17.01	15.000	20.000	17.13
5.000	8.000	5.90	5.000	8.000	6.13
5.000	8.000	5.59	5.000	8.000	5.18

Objective Function		
Total Cost (Rs.)	722527.79	
Cost (Rs./ton)	Period 1	Period 2
	24.4999611	23.66855809

solution for modified LP problem in development alternative with zero production at bench 3 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
2500	4500	0	1000	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
4500	6500	0	2500	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
4500	5623	0	1000	1000	2477	400	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12123									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.13	40.000	42.000	40.16
14.000	15.000	14.17	14.000	15.000	14.10
2.700	3.400	3.39	2.700	3.400	3.40
1.650	2.170	2.01	1.650	2.170	1.74
35.000	40.000	38.10	35.000	40.000	38.33
0.000	2.000	1.65	0.000	2.000	1.77
0.000	0.500	0.27	0.000	0.500	0.23
0.000	0.500	0.28	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.89	0.845	0.900	0.90
2.600	2.900	2.62	2.600	2.900	2.75
1.500	2.000	1.69	1.500	2.000	1.96
30.000	35.000	30.00	30.000	35.000	31.00
15.000	20.000	18.00	15.000	20.000	17.04
5.000	8.000	5.59	5.000	8.000	6.07
5.000	8.000	6.12	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	700191.91	
Cost (Rs./ton)	Period 1	Period 2
	23.00949824	23.66996274

solution for modified LP problem in development alternative with zero production at bench 3 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	4000	4500	0	1500	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	5500	6000	0	2500	2500	4000	1500	500	150
1712	3719	3731	1739	1674	1685	669	470	121	4731	5213	0	2367	1631	2000	972	0	0
15000									15000								
12575									13942								
15000									15000								
15519									16914								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.11	40.000	42.000	40.00
14.000	15.000	14.15	14.000	15.000	14.01
2.700	3.400	3.40	2.700	3.400	3.12
1.650	2.170	2.04	1.650	2.170	1.89
35.000	40.000	37.95	35.000	40.000	39.15
0.000	2.000	1.80	0.000	2.000	1.56
0.000	0.500	0.26	0.000	0.500	0.25
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.26
0.845	0.900	0.89	0.845	0.900	0.91
2.600	2.900	2.60	2.600	2.900	2.79
1.500	2.000	1.67	1.500	2.000	1.65
30.000	35.000	30.00	30.000	35.000	32.66
15.000	20.000	17.93	15.000	20.000	15.54
5.000	8.000	5.56	5.000	8.000	5.07
5.000	8.000	6.21	5.000	8.000	5.76

Objective Function		
Total Cost (Rs.)	833716.22	
	Period 1	Period 2
Cost (Rs./ton)	28.04875109	23.55672111

ution for modified LP problem in development alternative with zero production at bench 3 during periods 1 and 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
2500	4500	0	1000	1000	1500	400	0	0	4000	4500	0	1500	1000	2000	400	0	0
4500	6500	0	2500	2500	3000	1100	500	150	5500	6000	0	2500	2500	4000	1500	500	150
3763	4690	0	1946	1846	2107	437	210	0	5500	4500	0	2500	1000	2000	972	0	0
15000									15000								
12245									13500								
15000									15000								
15000									16472								

Quality Requirements						Objective Function		
Period 1			Period 2			Total Cost (Rs.)	741405.93	
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value			
40.000	42.000	40.26	40.000	42.000	40.00			
14.000	15.000	14.11	14.000	15.000	14.03			
2.700	3.400	3.33	2.700	3.400	3.20			
1.650	2.170	2.00	1.650	2.170	1.89			
35.000	40.000	38.08	35.000	40.000	38.94			
0.000	2.000	1.67	0.000	2.000	1.57			
0.000	0.500	0.27	0.000	0.500	0.25	Cost (Rs./ton)	Period 1	Period 2
0.000	0.500	0.28	0.000	0.500	0.26		23.87180526	23.27142959
100.000	100.000	100.00	100.000	100.000	100.14			
0.845	0.900	0.90	0.845	0.900	0.90			
2.600	2.900	2.64	2.600	2.900	2.76			
1.500	2.000	1.66	1.500	2.000	1.69			
30.000	35.000	31.43	30.000	35.000	32.02			
15.000	20.000	16.73	15.000	20.000	16.08			
5.000	8.000	5.45	5.000	8.000	5.27			
5.000	8.000	6.10	5.000	8.000	5.75			

solution for modified LP problem in development alternative with zero production at bench 4 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1500	3500	3000	0	1000	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
3750	5750	4000	0	2500	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2293	5750	3000	0	1048	2509	400	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
12091									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.12	40.000	42.000	40.16
14.000	15.000	14.06	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.74
35.000	40.000	38.30	35.000	40.000	38.33
0.000	2.000	1.73	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.68	2.600	2.900	2.75
1.500	2.000	1.84	1.500	2.000	1.96
30.000	35.000	30.99	30.000	35.000	31.00
15.000	20.000	16.93	15.000	20.000	17.04
5.000	8.000	5.89	5.000	8.000	6.07
5.000	8.000	5.61	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	709012.56	
	Period 1	Period 2
Cost (Rs./ton)	23.59754114	23.66996274

solution for modified LP problem in development alternative with zero production at bench 4 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2750	3250	4000	0	1000	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	4250	4750	5000	0	2500	4000	1500	500	150
1723	4500	3000	1000	1898	2479	400	0	0	2750	3734	4628	0	1000	2103	785	0	0
15000									15000								
12121									12112								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.21	40.000	42.000	40.00
14.000	15.000	14.10	14.000	15.000	14.08
2.700	3.400	3.40	2.700	3.400	3.34
1.650	2.170	1.84	1.650	2.170	1.67
35.000	40.000	38.14	35.000	40.000	38.79
0.000	2.000	1.77	0.000	2.000	1.63
0.000	0.500	0.26	0.000	0.500	0.21
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.81
1.500	2.000	1.85	1.500	2.000	2.00
30.000	35.000	31.09	30.000	35.000	31.01
15.000	20.000	16.96	15.000	20.000	16.96
5.000	8.000	5.90	5.000	8.000	6.03
5.000	8.000	5.60	5.000	8.000	5.09

Objective Function		
Total Cost (Rs.)	727432.64	
	Period 1	Period 2
Cost (Rs./ton)	24.24066733	24.25484182

ution for modified LP problem in development alternative with zero production at bench 4 during periods 1 and 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1500	3500	3000	0	1000	1500	400	0	0	2750	3250	4000	0	1000	2000	400	0	0
3750	5750	4000	0	2500	3000	1100	500	150	4250	4750	5000	0	2500	4000	1500	500	150
2293	5750	3000	0	1048	2509	400	0	0	2750	3734	4628	0	1000	2103	785	0	0
15000									15000								
12091									12112								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.12	40.000	42.000	40.00
14.000	15.000	14.06	14.000	15.000	14.08
2.700	3.400	3.40	2.700	3.400	3.34
1.650	2.170	1.84	1.650	2.170	1.67
35.000	40.000	38.30	35.000	40.000	38.79
0.000	2.000	1.73	0.000	2.000	1.63
0.000	0.500	0.26	0.000	0.500	0.21
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.68	2.600	2.900	2.81
1.500	2.000	1.84	1.500	2.000	2.00
30.000	35.000	30.99	30.000	35.000	31.01
15.000	20.000	16.93	15.000	20.000	16.96
5.000	8.000	5.89	5.000	8.000	6.03
5.000	8.000	5.61	5.000	8.000	5.09

Objective Function		
Total Cost (Rs.)	717785.81	
	Period 1	Period 2
Cost (Rs./ton)	23.59754592	24.25484137

solution for modified LP problem in development alternative with zero production at bench 5 during period 1

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1500	3500	3000	1000	0	1500	400	0	0	2000	2500	4000	1500	1000	2000	400	0	0
3750	5750	4000	2500	0	3000	1100	500	150	3000	3500	5000	2500	2500	4000	1500	500	150
2226	5750	3000	1000	0	2554	470	0	0	2000	3500	4000	1500	1000	2321	520	159	0
15000									15000								
11976									12000								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.20	40.000	42.000	40.16
14.000	15.000	14.09	14.000	15.000	14.10
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.85	1.650	2.170	1.74
35.000	40.000	38.01	35.000	40.000	38.33
0.000	2.000	1.91	0.000	2.000	1.77
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.27
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.75
1.500	2.000	1.84	1.500	2.000	1.96
30.000	35.000	31.08	30.000	35.000	31.00
15.000	20.000	16.95	15.000	20.000	17.04
5.000	8.000	5.89	5.000	8.000	6.07
5.000	8.000	5.62	5.000	8.000	5.28

Objective Function		
Total Cost (Rs.)	711680.41	
	Period 1	Period 2
Cost (Rs./ton)	23.77539455	23.66996588

solution for modified LP problem in development alternative with zero production at bench 5 during period 2

Quantity Requirements																	
Period 1									Period 2								
Bench					Additives				Bench					Additives			
1	2	3	4	5	Clay	Slate	Clay	Laterite	1	2	3	4	5	Clay	Slate	Clay	Laterite
1000	3000	3000	1000	1000	1500	400	0	0	2500	3000	4000	1500	0	2000	400	0	0
2500	4500	4000	2500	2500	3000	1100	500	150	4250	4750	5000	2500	0	4000	1500	500	150
1723	4500	3000	1000	1898	2479	400	0	0	2500	3986	4000	1500	0	2000	580	434	0
15000									15000								
12121									11986								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			Period 2		
Minimum	Maximum	Blend Value	Minimum	Maximum	Blend Value
40.000	42.000	40.21	40.000	42.000	40.24
14.000	15.000	14.10	14.000	15.000	14.14
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.84	1.650	2.170	1.73
35.000	40.000	38.14	35.000	40.000	38.19
0.000	2.000	1.77	0.000	2.000	1.81
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.76
1.500	2.000	1.85	1.500	2.000	1.97
30.000	35.000	31.09	30.000	35.000	31.09
15.000	20.000	16.96	15.000	20.000	17.07
5.000	8.000	5.90	5.000	8.000	6.09
5.000	8.000	5.60	5.000	8.000	5.26

Objective Function		
Total Cost (Rs.)	713377.87	
Cost (Rs./ton)	Period 1	Period 2
	24.240667	23.31785797

ution for modified LP problem in development alternative with zero production at bench 5 during periods 1 and 2

Quantity Requirements																	
Period 1									**Period 2**								
Bench					**Additives**				**Bench**					**Additives**			
1	**2**	**3**	**4**	**5**	**Clay**	**Slate**	**Clay**	**Laterite**	**1**	**2**	**3**	**4**	**5**	**Clay**	**Slate**	**Clay**	**Laterite**
1500	3500	3000	1000	0	1500	400	0	0	2500	3000	4000	1500	0	2000	400	0	0
3750	5750	4000	2500	0	3000	1100	500	150	4250	4750	5000	2500	0	4000	1500	500	150
2226	5750	3000	1000	0	2554	470	0	0	2500	3986	4000	1500	0	2000	580	434	0
15000									15000								
11976									11986								
15000									15000								
15000									15000								

Quality Requirements					
Period 1			**Period 2**		
Minimum	**Maximum**	**Blend Value**	**Minimum**	**Maximum**	**Blend Value**
40.000	42.000	40.20	40.000	42.000	40.24
14.000	15.000	14.09	14.000	15.000	14.14
2.700	3.400	3.40	2.700	3.400	3.40
1.650	2.170	1.85	1.650	2.170	1.73
35.000	40.000	38.01	35.000	40.000	38.19
0.000	2.000	1.91	0.000	2.000	1.81
0.000	0.500	0.26	0.000	0.500	0.23
0.000	0.500	0.29	0.000	0.500	0.26
100.000	100.000	100.00	100.000	100.000	100.00
0.845	0.900	0.90	0.845	0.900	0.90
2.600	2.900	2.69	2.600	2.900	2.76
1.500	2.000	1.84	1.500	2.000	1.97
30.000	35.000	31.08	30.000	35.000	31.09
15.000	20.000	16.95	15.000	20.000	17.07
5.000	8.000	5.89	5.000	8.000	6.09
5.000	8.000	5.62	5.000	8.000	5.26

Objective Function		
Total Cost (Rs.)	706398.79	
	Period 1	**Period 2**
Cost (Rs./ton)	23.77539541	23.31785715